Convex Optimization

Practical session # 1

October 2, 2025

Installing CVXPY

- If you have Python installed on your computer, then you may follow the instructions at https://www.cvxpy.org/install/index.html (e.g. by running `pip install cvxpy`)
- If you prefer to compile Python without having it installed it locally, then you may use https://jupyter.org/. Go to "Try" → "JupyterLab", and there you may create a new "Notebook". The website is free and requires no registration.

In this practical we went through some basic examples on how to use CVXPY - my code is attached in a separate document PDF.

Exercises / Demonstrations

1. Get familiar with basic Python commands (print 'Hello world')
2. Solve the linear program

$$\begin{aligned} \text{minimize } & f(x,y) = x + y \\ \text{subject to } & 2x + y \geq 2 \\ & x + 2y \geq 2 \end{aligned}$$

   both by hand, and by using CVXPY - compare the results.

   Observe how we introduce variables, objective functions, constraints, and optimization problems in CVXPY.

3. Try for yourself to solve the problem

$$\begin{aligned} \text{minimize } & f(x,y) = 4x + 5y + 3z \\ \text{subject to } & x^2 + 2y^2 + z^2 \leq 4, \end{aligned}$$

   using CVXPY.
   (*) Try computing the exact solution by hand - how does it compare to the CVXPY solution?

4. Consider the problem of the 'optimal diet', from the first lecture:

$$\begin{aligned} \text{minimize } & f(x) = c^T x \\ \text{subject to } & Ax \succeq b \\ & x \succeq 0, \end{aligned}$$

   where

$$A = \begin{bmatrix} 35 & 0.5 & 0.5 \\ 60 & 300 & 10 \\ 30 & 20 & 10 \end{bmatrix}, b = \begin{bmatrix} 0.5 \\ 15 \\ 4 \end{bmatrix}, c = \begin{bmatrix} 0.75 \\ 0.5 \\ 0.15 \end{bmatrix}$$

   Solve this problem using CVXPY, specifically getting familiar with how vectors/matrices are encoded in NumPy / CVXPY.

5. Come up with an easy example of a linear program (or other convex problem), such that

- it has no solution,
- it has an optimal solution that is not unique,
- it is unbounded.

What does CVXPY output as optimal value, optimal solution, and 'status' of these problems?

6. Have a look at the documentation of CVXPY https://www.cvxpy.org/tutorial/index.html. In particular, check which constraint and objective functions can be encoded (https://www.cvxpy.org/tutorial/functions/index.html).

Remark

The main advantages of CVXPY are that:

- It is free
- it is relatively intuitive to use (using notation that is close to mathematical notation)
- it can handle small and medium sized problems relatively quickly.
- It was co-developed by Boyd, and therefore aligns well with the material of this course.

However, depending on the application, other software might be more helpful. There is a very big variety of optimization software, both for free and commercial use - see e.g. https://en.wikipedia.org/wiki/List_of_optimization_software. Another prominent optimization package for Python is SciPy, which includes more general-purpose tools that can also be used to solve some non-convex optimization problems; this comes however at the cost of more manual calibrations and low-level understanding of algorithms.

Also mathematics software that you might be already familiar with (MATLAB, Matematica, Maple) has built in optimization tools - we are not going to discuss them in this course - however, feel invited to experiment with them and share what you think are their advantages/disadvantages.