Convex Optimization 2025/26

Homework # 5
December 11, 2025

Instructions

- This homework counts towards the credit, if you did not already score enough points on one of the previous 4 assignments.
- Please, submit your homeworks to kompatscher@karlin.mff.cuni.cz. The subject of your email should start with `[Convex Optimization]`.
- The written solutions are expected to be submitted in a single `.pdf` file and include your name. The use of LaTeX is encouraged - if you use handwriting, make sure it's legible. Additionally attach any Python code you used.
- Please, send your submissions no later than January 7, 15:40.

Exercise 1 (10 points) The function $f(x) = \log(e^x + e^{-x})$ has a unique minimum at $x^* = 0$. Run Newton's method with fixed step size $t = 1$, and starting points $x^{(0)} = 1$ and $x^{(0)} = 1.1$, and compute the first few iterations. What do you observe? Compare with the graph of $f'(x)$.

Exercise 2 (10 points) Let us consider an unconstraint convex problem (*minimize* $f(x)$).

The *coordinate descent method* is the descent method, in which we only consider cardinal directions as descent directions, i.e. $\Delta x = \pm e_i$, for some coordinate $i \in \{1, 2, \dots, n\}$.

Find a convex function $f(x)$ and a start point $x^{(0)}$, such that coordinate descent (with exact line-search) gets stuck at a different point than the optimal solution x^*. (Hint: experiment with $n = 2$ and piece-wise linear functions.)

Exercise 3 (20 points)

In this problem, you will compute a test for the detection of counterfeit banknotes via stochastic vector machines.

- The data can be found on https://archive.ics.uci.edu/ml/machine-learning-databases/00267/data_banknote_authentication.txt, a text file with 1372 lines. Each row contains four numbers coming from a 'Wavelet transformation' of a scan of a banknote (something roughly similar to a Fourier transform, more details are available at https://archive.ics.uci.edu/ml/datasets/banknote+authentication#) and a single number 0 or 1 that indicates whether the banknote was counterfeit.
- Overwrite these classifier values 0 and 1, with 1 and -1 such that the data is consistent with our lecture (at the end, there should be 762 rows with value -1, corresponding to the counterfeits).
- Next, we split the data into a training and testing set. So create two lists `train` and `test`. The list `test` should contain every 5th entry, and `train` all other lines (i.e. `train` has 1098 rows and `test` has 274 rows).
- Using CVXPY, compute linear discriminators $f(x) = a^T x + b$ for the training data in `train`. Do this for at least five different values of the regularization parameter $\gamma > 0$.
- Finally, compare the resulting classfier with the data in `test`. For which values of γ do you obtain the highest success rate?