Convex Optimization 2025/26

Homework # 4
November 27, 2025

Instructions

- Please, submit your homeworks to kompatscher@karlin.mff.cuni.cz. The subject of your email should start with `[Convex Optimization]`.
- The written solutions are expected to be submitted in a single `.pdf` file and include your name. The use of LaTeX is encouraged - if you use handwriting, make sure it's legible. Additionally attach any Python code you used.
- There will be 4 homework assignments, on each of which you need to score at least 26 out of 40 points to obtain the credit (zápočet) for the course.
- Please, send your submissions no later than December 11, 15:40.

Exercise 1 (5 points) Consider the unfeasible LP

$$\begin{aligned} &\text{minimize } x \\ &\text{subject to } x \leq 0 \\ &\qquad\qquad 1 \leq 0 \end{aligned}$$

Compute its dual problem and show that it is also unfeasible, i.e. $p^* = \infty$, and $d^* = -\infty$. What does this tell us about the (strong) duality for LPs?

Exercise 2 (10 points) What is the solution of the norm approximation problem with one scalar variable $x \in \mathbb{R}$:

$$\text{minimize} \qquad \|x\mathbf{1} - b\|$$

for ℓ_1-, ℓ_2-, and ℓ_∞-norms? Here, $\mathbf{1} = (1, \ldots, 1) \in \mathbf{R}^n$ and $b \in \mathbf{R}^n$ is a constant vector.

Exercise 3 (10 points) Let $M > 0$. Show that the Huber penalty function

$$\phi(u) = \begin{cases} u^2 & \text{if } |u| \leq M \\ M(2|u| - M) & \text{else,} \end{cases}$$

is equal to $\phi(u) = \inf_{x \in \mathbf{R}}(2M|x| + (x-u)^2)$.

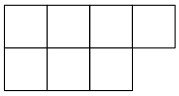
Use this to show that Huber penalty approximation problem

$$\begin{aligned} &\text{minimize } \sum_{i=1}^{m} \phi(r_i) \\ &\text{subject to } r = Ax - b \end{aligned}$$

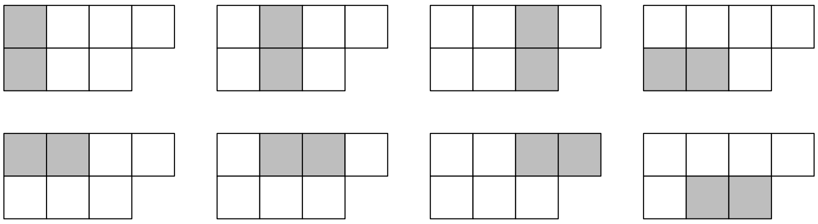
is equivalent to a quadratic program.

Exercise 4 (15 points)

Consider the following version of the game of Battleship: There is just one round. Player 1 places a 1×2 ship somewhere in the irregular playing field which looks like in the picture



with 8 possible ship positions:



Player 2, not knowing Player 1's choice, picks one of the 7 squares to shoot at. If Player 2's shot hits the ship, Player 1 loses a point and Player 2 gains a point, otherwise Player 1 gains a point and Player 2 loses a point.

Use `cvxpy` to calculate the worst-case optimal strategy for both players and the value of this game. You do not have to send your code, if you explain what your optimization problem was and how it connects to the game.