Convex Optimization

Practical session # 10

December 4, 2024

Consider a linear measurement model,

$$y_i = a_i^T x + v_i$$

where we observe (y_i, a_i) and are trying to estimate x_i (v_i is the measurement error or noise). As usual we assume v_i are independent identically distributed (IID) with density p. The likelihood function is then

$$p_x(y, a) = \prod_{i=1}^{m} p(y_i - a_i^T x)$$

and so the log-likelihood function is

$$\ell(x) = \log p_x(y, a) = \sum_{i=1}^{m} \log p(y_i - a_i^T x).$$

Exercise 1

Show how to reduce the ML estimate problem to an LP problem when the noise is *exponentially distributed*, i.e.

$$p(z) = \begin{cases} (1/a)e^{-z/a}, & z \geq 0 \\ 0, & z < 0, \end{cases}$$

where $a > 0$.

Exercise 2

Suppose that the noise density $p(z)$ corresponds to the uniform distribution on $[-\alpha, \alpha]$:

$$p(z) = \begin{cases} 1/(2\alpha), & |z| \leq \alpha \\ 0, & |z| \geq \alpha, \end{cases}$$

(a) Find all ML estimates for fixed $y \in \mathbb{R}^m$ and $A \in \mathbb{R}^{n \times m}$.

(b) Assume now that α is not known, so we wish to estimate it along with x. Show that the ML estimations of x and α are found by solving the ℓ_∞-norm optimization problem

$$\text{minimize} \qquad \|Ax - y\|_\infty,$$

where a_i^T are the rows of A.

Exercise 3

Give the explicit solution of the following LP:

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & \mathbf{1}^T x = 1, \quad x \succeq 0. \end{array}$$

Exercise 4

Let

$$P = \begin{pmatrix} 0.5 & 0.1 \\ 0.3 & 0.4 \\ 0.1 & 0.6 \\ 0.1 & 0.1 \end{pmatrix}$$

Compute determenistic detectors associated with the weight matrices $W = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$ and $W' = \begin{pmatrix} 0 & 3 \\ 1 & 0 \end{pmatrix}$.