Universal Algebra Exercises - Homework 2

Exercise 2.1. We call an algebra k-supernilpotent if every $k+1$-ary absorbing polynomial is constant. Let $\mathbb{A} = (\mathbb{Z}_3 \times \mathbb{Z}_2, +, (0,0), -, f)$, where f is the unary function defined by

$$f(x,y) = \begin{cases} (0,1) & \text{if } x = 0 \\ (0,0) & \text{otherwise.} \end{cases}$$

Show that this algebra is 2-nilpotent but not k-supernilpotent for any k.

Exercise 2.2. Let L be a distributive lattice. Prove that $(c,d) \in \mathrm{Cg}^L(a,b)$ if and only if

$$c \wedge (a \wedge b) = d \wedge (a \wedge b) \quad \text{and} \quad c \vee (a \vee b) = d \vee (a \vee b)$$

Conclude that the variety of distributive lattice has definable principal congruences.

Exercise 2.3. Let $\mathbb{A}$ be a finite nilpotent Maltsev algebra. Show that $\mathrm{HSP}(\mathbb{A})$ is residually finite if and only if $\mathbb{A}$ is abelian. (Hint: remember that Maltsev algebras are congruence modular and use Theorem 2.24.)