Universal Algebra 2 - Exercises 6

Exercise 5.4. Recall the structure $\mathbb{A} = (\{0,1\}; R_{000}, R_{001}, R_{011}, R_{111})$ and that $\mathrm{CSP}(\mathbb{A}) = 3\mathrm{SAT}$. Show that all polymorphisms of $\mathbb{A}$ are projections. (Hint: what can you *pp*-define from the relations in $\mathbb{A}$?)

Exercise 6.1. Let $\mathbb{A}$ and $\mathbb{B}$ be two homomorphically equivalent relational structures ($\mathbb{A} \to \mathbb{B}$ and $\mathbb{B} \to \mathbb{A}$). Show that there is a minion homomorphism $\mathrm{Pol}(\mathbb{A}) \xrightarrow{\text{minion}} \mathrm{Pol}(\mathbb{B})$.

Exercise 6.2. Let $\mathbb{A} = (\{0\}, =)$ and $\mathbb{B} = (\{0,1\}, \leq)$. Show that $\mathbb{A}$ and $\mathbb{B}$ are homomorphically equivalent but that there is no clone homomorphism $\mathrm{Pol}(\mathbb{A}) \xrightarrow{\text{clone}} \mathrm{Pol}(\mathbb{B})$. (Hint: show that $\mathrm{Pol}(\mathbb{B})$ does not contain a Maltsev term)

Exercise 6.3. Let $R \subseteq A^n$ be a relation compatible with a majority polymorphisms $m : A^3 \to A$.

$$m(x,x,y) \approx m(x,y,x) \approx m(y,x,x) \approx x$$

Denote by $\pi_{i,j}(R)$ the projections of R to the coordinates i, i $(1 \leq i, j \leq n)$.

$$\pi_{i,j} = \{(a_i, a_j) \mid (a_1, \ldots, a_n) \in R\}$$

Show that R is determined by these binary projections, i.e.

$$(a_1, \ldots, a_n) \in R \iff \forall i,j \quad (a_i, a_j) \in \pi_{i,j}(R)$$

Conclude that R is *pp*-definable from binary relations.

Exercise 6.4. Find a finite set of relations $\{R_1, \ldots, R_n\}$ on the set $\{0,1\}$ such that $\mathrm{Pol}(R_1, \ldots, R_n)$ is the clone generated by the unique majority operation on $\{0,1\}$.