Universal Algebra 2 - Exercises 5

Exercise 4.2. Let $\mathbb{A}$ and $\mathbb{B}$ be two algebras of finite type on the same domain with $\text{Clo}(\mathbb{A}) = \text{Clo}(\mathbb{B})$. Show that $\mathbb{A}$ is finitely based if and only if $\mathbb{B}$ is finitely based. Is this still true if we do not assume finite type?

Exercise 5.1. Let $\mathbb{A}$ be a relational structure in signature τ. Show that the following decision problem is equivalent to $\text{CSP}(\mathbb{A})$:

- INPUT: a τ structure $\mathbb{X}$
- QUESTION: is there a homomorphism $\mathbb{X} \to \mathbb{A}$?

Conclude that if there are homomorphisms $\mathbb{A} \to \mathbb{B}$ and $\mathbb{B} \to \mathbb{A}$, then the CSPs of $\mathbb{A}$ and $\mathbb{B}$ are the same.

Exercise 5.2. Consider the computational problem nCOLOR, of coloring a given graph with n many colors.

- Find a relational structure $\mathbb{A}$ such that $n\text{COLOR} = \text{CSP}(\mathbb{A})$.
- Find a polynomial time reduction of 3COLOR to nCOLOR and conclude that nCOLOR is NP-hard.

Exercise 5.3. Let A be a finite set. Show that a function $f : A^n \to A$ preserves all relations on A if and only if it is a projection.

Exercise 5.4. Recall the structure $\mathbb{A} = (\{0,1\}; R_{000}, R_{001}, R_{011}, R_{111})$ and that $\text{CSP}(\mathbb{A}) = \text{3SAT}$. Show that all polymorphisms of $\mathbb{A}$ are projections. (Hint: what can you *pp*-define from the relations in $\mathbb{A}$?)