Universal Algebra 2 - Exercises 4

Exercise 3.2. Prove that the polynomial equivalence problem of nilpotent rings is solvable in polynomial time. Here is the strategy:

- Convince yourself that a commutative ring R is k-step nilpotent if and only if $x_1 \cdot \dots \cdot x_{k+1} \approx 0$ holds.
- Given a polynomial p in R, show that $p \approx 0$ if and only if $p(\bar{a}) = 0$ for all tuples $\bar{a}$ with at most k many non-0 entries.
- Conclude that $\mathrm{PolEQV}(R)$ is solvable in polynomial time.

Exercise 4.1. Recall that a variety is already finitely based, if it has definable principal congruences and finitely many subdirectly irreducibles (up to isomorphism). Consider commutative rings R that satisfy equation $x^n \approx x$.

- Show that every subdirecly irreducible such ring is a field of order d, where $d - 1 \mid n - 1$.
- Conclude that $\mathrm{HSP}(R)$ is finitely based.

Exercise 4.2. Let $\mathbb{A}$ and $\mathbb{B}$ be two algebras of finite type on the same domain with $\mathrm{Clo}(\mathbb{A}) = \mathrm{Clo}(\mathbb{B})$. Show that $\mathbb{A}$ is finitely based if and only if $\mathbb{B}$ is finitely based. Is this still true if we do not assume finite type?