Universal Algebra 2 - Exercises 3

Exercise 2.5. Show the following properties of the centralizer relation C:

- $C(\alpha, \beta; \alpha)$ and $C(\alpha, \beta; \beta)$
- Let Γ be a set of congruences. If $C(\alpha, \beta; \gamma)$ for all $\gamma \in \Gamma$, then $C(\alpha, \beta; \bigwedge \Gamma)$.

Exercise 3.1. Consider the Loop $(L, \cdot)$ with universe $\mathbb{Z}_4 \times \mathbb{Z}_2$ given by the multiplication

$$(a, b) \cdot (c, d) = (a + c, b + d) \quad \text{unless } b = d = 1$$
$$(a, 1) \cdot (c, 1) = (a * c, 0) \quad \text{where}$$

$*$	0	1	2	3
0	1	0	2	3
1	0	2	3	1
2	2	3	1	0
3	3	1	0	2

Consider the map $f : L \to \mathbb{Z}_2, (a, b) \mapsto b$, its kernel α and the α-block N of $(0, 0)$.

- Show that f is a homomorphism and that N is an abelian subloop.
- Show that α is not an abelian congruence, i.e. $C(\alpha, \alpha, 0)$ does not hold.

Exercise 3.2. Prove that the polynomial equivalence problem of nilpotent rings is solvable in polynomial time. Hint: Look at Example 2.26 in the script.

Exercise 3.3. We call an algebra k-supernilpotent if every $k+1$-ary absorbing polynomial is constant. Consider the algebra $(\mathbb{Z}_9, +, 0, -, f_n(x_1, \ldots, x_n) \mid n \in \mathbb{N})$ where $f_n(x_1, \ldots, x_n) = 3 \cdot x_1 \cdot \cdots \cdot x_n$. Show that this algebra is 2-nilpotent but no k supernilpotent for any k