FACULTY
OF MATHEMATICS
AND PHYSICS
Charles University

# Book of Abstracts

of the

## 8th Day of Doctoral Students
## of the School of Mathematics

## June 29, 2022

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http://www.karlin.mff.cuni.cz/~knobloch/DDS-M/2022/

# Preface

In the beginning of 2014, the Management of the Faculty of Mathematics and Physics decided that the traditional conference of PhD students called the WDS (Week of Doctoral Students) would not be organized as an activity of the entire faculty. Instead, the decision as to whether to organize the conference or not was left to the respective Schools (of Computer Science, of Mathematics, and of Physics).

Since then, the School of Mathematics organized this event as WDS-M (Week of Doctoral Students of the School of Mathematics). Except for 2014, WDS-M was always a one-day conference and therefore, the new name DDS-M (Day of Doctoral Students of the School of Mathematics) was introduced in the last year. Since WDS-M was not organized in 2020 due to the COVID-19 Pandemic, the conference of PhD students at the School of Mathematics is organized for the 8th time in this year, see also `http://www.karlin.mff.cuni.cz/~knobloch/DDS-M/2022/`. The original WDS continued at the School of Physics in its 31st edition (June 7–9, 2022) as a conference for PhD students of physical study programs, see `http://www.mff.cuni.cz/veda/konference/wds/`.

This year, 13 students have registered as active participants to the conference. We believe that this event, which takes place in the "mathematical" Karlín building of the faculty, will attract the attention of the students but also of the broad mathematical audience. We thus encourage all of those interested in the scientific activities of our doctoral students to attend this meeting.

The conference is co-organized by the *School of Mathematics, Faculty of Mathematics and Physics, Charles University*, and *Charles University Chapter of SIAM*.

Prague, June 29, 2022

doc. Mgr. Petr Knobloch, Dr., DSc.
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# Contents

## On Enochs' conjecture

**M.Sc. Asmae Ben Yassine**

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*Study branch:* P4M1
*Year of study:* 3
*Supervisor:* Prof. Dr. Jan Trlifaj

### Abstract

Enochs' conjecture says that each covering class of modules is closed under direct limits. The conjecture is still open in its full generality, we will present several important special cases where it holds true.

# Quadratic forms and Pythagoras numbers over number fields

**Mgr. Ester Sgallová**

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**Abstract**

This talk is on the topic of the universal quadratic forms and Pythagoras numbers over number fields. The most famous universal quadratic form over Q appears in Lagrange's four-squares theorem, which states that every positive integer in Z is a sum of four squares. However, we will focus on quadratic forms over quadratic and biquadratic fields and their connection to the Pythagoras number, i.e. the smallest number n such that any sum of squares is, in fact, a sum of n squares. We will examine the Pythagoras number of the ring of integers in a totally real biquadratic number field.

# Arithmetics of number fields

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## Abstract

Various diophantine equations naturally lead to considering field extensions of $\mathbb{Q}$ whose degree is finite, called algebraic number fields. The role of integers in a number field $K$ is played by its ring of integral elements. However, unique factorization fails in general: it can happen that an algebraic integer factors as a product of irreducible elements in more than one way. How far a given field is from having the unique factorization property is measured by the class number, which is quite an elusive quantity even for the simplest number fields. The class number problem, which asks how many real quadratic fields have class number one, is the most important unsolved problem in the area. We will introduce the discriminant, the class number, and several other invariants and see how they are connected by the class number formula.

# Quantum groups and q-determinant

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## Abstract

In this talk, we would discuss some basics of Quantum groups and focusing on algebraic version of quantum groups (quantum algebras), we would discuss some finite dimensional representation theory. We would also define q-determinant for Drinfeld-Jimbo type quantum groups.

# A variational approach to hyperbolic evolutions

**Mgr. Antonín Češík**

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**Abstract**

This talk is about a variational time-stepping minimization scheme for solving hyperbolic (second-order in time) problems, introduced by B. Benešová, M. Kampschulte, S. Schwarzacher in 2020. It is an extension of the famous minimizing movements method by De Giorgi. Due to its variational nature, the scheme is applicable to highly non-linear problems. There are many applications in dynamical problems of continuum mechanics. Notably in fluid-structure interaction, as shown in several recent works, mainly by S. Schwarzacher, M. Kampschulte, B. Benešová and D. Breit.

I will talk about two of my current projects. Firstly, the problem of (self-)collisions of (visco-)elastic solids (with M. Kampschulte and G. Gravina), secondly investigating the convergence properties of this scheme (with S. Schwarzacher).

## Homeomorphism equivalence relation

**Mgr. Jan Dudák**

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### Abstract

The homeomorphism equivalence relation restricted to various classes of metrizable compact spaces can be studied using methods of invariant descriptive set theory. Namely, the notion of a Borel reduction allows us to determine the complexity of a given homeomorphism classification problem by comparing it to benchmark classification problems coming from various fields of mathematics (e.g. from order theory, graph theory, algebra or functional analysis).

# Mixed precision GMRES-based iterative refinement with recycling

**M.Sc. Eda Oktay**

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*Supervisor:* Erin Carson, Ph.D.

## Abstract

Krylov subspace recycling is a well-known technique for reusing information across sequential invocations of a Krylov subspace method on systems with the same or a slowly changing coefficient matrix. In this talk, we present a mixed precision GMRES-based iterative refinement solver incorporated with Krylov subspace recycling approach. The insight in this algorithm is that in each refinement step, we call preconditioned GMRES on a linear system with the same coefficient matrix, with only the right-hand side changing. In this way, the GMRES solves in subsequent refinement steps can be accelerated by recycling information obtained from the first step. After giving a background on GMRES-based iterative refinement and Krylov subspace recycling, we present numerical experiments that show the advantage of combining this approach.

# An introduction and numerical approximation of the $\star$-product

**M.Sc. Shazma Zahid**

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### Abstract

Simulating the dynamics of spins is fundamental for nuclear magnetic resonance (NMR). Mathematically, such systems can be represented by coupled first-order differential equations with generally non-constant coefficients associated with very large sparse matrices. The solution of such systems (also known as the time-ordered exponential) is a computationally challenging problem arising in a variety of other applications. In this talk, we present a convolution-like product known as $\star$-product, which provides an alternative way to solve the system under consideration. An accurate approximation of such a product might be key to the efficient numerical computation of large-to-huge time-ordered exponentials. After introducing the $\star$-product, we show how rectangular quadrature and trapezoidal rules are used to numerically approximate the $\star$-product resulting in a usual matrix product in the case of the rectangular quadrature, whereas work is still in progress for the latter case.

# Crofton's formula – a wonderful way how to connect seemingly unrelated problems in random geometry

**Mgr. Dominik Beck**

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## Abstract

A classical problem in random geometry goes like this: In a given cube of unit volume, what is the mean distance between two points selected randomly uniformly within its interior? Of course, the answer can be rewritten as solving a particular six-fold integral containing square root. But can we say indeed, looking at the integral, that we can express its value in an exact form (in terms of just elementary functions)? The answer is surprisingly yes and its exact form is equal to

$$\frac{4}{105}+\frac{17\sqrt{2}}{105}-\frac{2\sqrt{3}}{35}-\frac{\pi}{15}+\frac{1}{5}\operatorname{arccoth}\sqrt{2}+\frac{4}{5}\operatorname{arccoth}\sqrt{3},$$

the value nowadays known as the so called Robbins constant. The result is mesmerising in many ways – mainly because of the fact that the mentioned six-fold integral cannot be solved directly. One successful and elegant way how to approach the problem indirectly is to use the Crofton's reduction technique. Using his eponymous formula, we can relate our original problem to more elementary ones which are easily solvable in exact form. The main goal of this particular lecture is to introduce the audience with the Crofton's formula and to demonstrate its use on a few examples to derive some exact and unexpected results in the field of random geometry.

# Statistical assessment of grain interactions in polycrystalline materials

**Mgr. Iva Karafiátová**

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**Abstract**

Stochastic models are frequently used in the field of modelling of polycrystalline materials. The overall structure itself is usually modelled by tessellations. Our focus lies mainly on understanding advanced grain characteristics such as stress within grains, grain lattice orientations and normals of grain boundaries. These characteristics play an essential role in the mechanical properties of the material.

We consider a random marked tessellation in which the marks are crystal lattice orientations, stress tensors, volumes etc. There exist well-approximating methods for modelling the unmarked tessellation. The aim is to extend these methods to fit a stochastic model to the real microstructure, i.e. the marked tessellation, in which the marks are modelled conditionally on the underlying tessellation. The model should take into account spatial dependencies, especially among the lattice orientations, which are often present. To achieve this, one needs to be able to measure and test the overall dependence of the orientations of neighbouring grains. This is the essential step of the current research.

# Applying monoid duality to interacting particle systems

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**Abstract**

In the study of interacting particle systems duality is an important tool used to prove various types of long-time behavior, for example convergence to an invariant distribution. The two most used types of dualities are additive and cancellative dualities, which we are able to treat in a unified framework considering commutative monoids (i.e. semigroups containing a neutral element) as cornerstones of such a duality. For interacting particle systems on local state spaces with more than two elements this approach revealed formerly unknown dualities.

## Copula-based trading of cointegrated cryptocurrency pairs

**Mgr. Masood Tadi**

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### Abstract

Pairs trading is a well-known statistical arbitrage strategy used in the financial market to detect pricing anomalies among financial assets using statistical techniques. This research aims to develop a pairs trading strategy in two steps (formation and trading) and reproduce these steps several times from 27-09-2018 to 30-12-2021. In the first step, we gather multiple cryptocurrencies' historical minute-by-minute closed prices and discover nonlinear cointegrated pairs using the Kapetanios-Snell-Shin (KSS) test. Among them, the nominated pair is selected with the highest Kendall's Tau correlation. Then, an appropriate bivariate copula family is specified to demonstrate the dependence structure of two assets, and the corresponding parameters are obtained by canonical maximum likelihood estimation. In the second step, the information from the dependence structure is used to determine trading signals for the crypto coins that are overvalued or undervalued compared to their pairs. The strategy is backtested by generating trading signals and executing the simulated trades.

# Posters

## Mean distance in convex polyhedra

**Mgr. Dominik Beck**

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### Abstract

Mean distance between two randomly selected points chosen uniformly from the interior of a given convex polyhedron is derived for various polyhedra using a modification of the Crofton Reduction Technique. It is shown that the method can be easily extended to find the exact value of the mean distance in any convex polyhedra in general.

## Computerized adaptive testing

**Mgr. Iván Pérez**

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