FACULTY
OF MATHEMATICS
AND PHYSICS
Charles University

Book of Abstracts

of the

7th Day of Doctoral Students
of the School of Mathematics

June 10, 2021

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LaTeX editing by Petr Knobloch

http://www.karlin.mff.cuni.cz/~knobloch/DDS-M/2021/

Preface

In the beginning of 2014, the Management of the Faculty of Mathematics and Physics decided that the traditional conference of PhD students called the WDS (Week of Doctoral Students) would not be organized as an activity of the entire faculty. Instead, the decision as to whether to organize the conference or not was left to the respective Schools (of Computer Science, of Mathematics, and of Physics).

Since then, the School of Mathematics organized this event as WDS-M (Week of Doctoral Students of the School of Mathematics). Except for 2014, WDS-M was always a one-day conference and therefore, the new name DDS-M (Day of Doctoral Students of the School of Mathematics) was introduced in this year. Since WDS-M was not organized in 2020 due to the COVID-19 Pandemic, the conference of PhD students at the School of Mathematics is organized for the 7th time in this year, see also `http://www.karlin.mff.cuni.cz/~knobloch/DDS-M/2021/`. The original WDS continues at the School of Physics in its 30th edition as a conference for PhD students of physical study programs, see `http://www.mff.cuni.cz/veda/konference/wds/`.

This year, 24 students have registered as active participants to the conference. We believe that this event, which takes place in the "mathematical" Karlín building of the faculty, will attract the attention of the students but also of the broad mathematical audience. We thus encourage all of those interested in the scientific activities of our doctoral students to attend this meeting.

The conference is co-organized by the *School of Mathematics, Faculty of Mathematics and Physics, Charles University*, and *Charles University Chapter of SIAM*.

Prague, June 10, 2021

doc. Mgr. Petr Knobloch, Dr., DSc.
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Contents

Description of K-spaces by means of J-spaces and the reverse problem in the limiting real interpolation

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Abstract

We establish conditions under which K-spaces in the limiting real interpolation involving slow varying functions can be described by means of J-spaces and we also solve the reverse problem. To this end, we prove several versions of the fundamental lemma of the real interpolation theory. We apply our results to obtain density theorems for the corresponding limiting interpolation spaces.

Fluid structure interaction in an incompressible conducting fluid

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Abstract

We study a model describing the motion of an insulating rigid body inside of an incompressible, electrically conducting fluid, governed by the Navier-Stokes equations and the Maxwell system. We show the existence of a weak solution to this model by approximating it through a regularized and discretized system. The result is joint work with Barbora Benešová, Šárka Nečasová and Anja Schlömerkemper.

Weak* derived sets

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Abstract

In this talk we recall the basic concepts of reflexive and quasi-reflexive Banach spaces and introduce the notion of a weak* derived set, that is the set of all limits of bounded weak* convergent nets from a given set. We then show how the reflexivity and quasi-reflexivity of a Banach space relates to the behaviour of weak* derived sets of some classes of sets in its dual space. In the end, we briefly mention some results concerning iterated weak* derived sets.

Fine properties of Sobolev functions in the context of rearrangement-invariant spaces

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Abstract

Let Ω be a domain in the Euclidean space $\mathbb{R}^n$ having the outer cone property and let $d(x) = \operatorname{dist}(x, \partial\Omega)$. Given $p \in [1, \infty)$ and a scalar function u of several variables, we seek minimal requirements on the regularity of the function u/d in order that u from $W^{1,p}$ belongs to the Sobolev space $W_0^{1,p}$ sheltering functions with zero boundary traces. We present a new such condition in terms of Lorentz spaces. We will further present another recent collection of results, originally also motivated by investigation of fine properties of traces of Sobolev functions. Considered in a broader perspective of Sobolev embeddings into spaces furnished with Frostman measures (that is, measures characterized by the rate of their decay on shrinking balls), the trace problem leads to interesting questions concerning certain new scale of function spaces. Such spaces are determined by the functional

$$\|f\|_{X^{\langle\alpha\rangle}} = \|((|f|^{\alpha})^{**})^{\frac{1}{\alpha}}\|_{\overline{X}},$$

defined on measurable functions, in which α is a positive real parameter, and $\overline{X}$ is the representation space of certain rearrangement-invariant space X. We will survey a variety of results concerning these spaces including their relations to customary function spaces, their mutual embeddings and duality properties. We discover a new one-parameter path of function spaces leading from a Lebesgue space to a Zygmund class independent of the classical one.

Hydrodynamic stability for the dynamic slip boundary condition

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Abstract

We consider the incompressible Navier-Stokes equation with the dynamic slip boundary condition. Our first goal is to prove the so-called linearization principle in the class of weak solutions satisfying the energy inequality. By this we mean that if the spectrum of certain operator has only positive real parts, then the stationary solution u^* of the Navier-Stokes equation is stable with respect to sufficiently small initial perturbations. We deal further with two explicit geometries, namely with either two infinite parallel planes or two concentric cylinders, where the solution u^* corresponds either to Couette/Poiseuille or Taylor-Couette flow. We eventually compare our results with well-known analogue results in the case of Dirchlet boundary condition.

Numerical solution of inverse problems in 3D imaging

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Abstract

In this research we study discrete inverse problems $A\,x \sim b$ arising in 3D imaging with a specific focus on cryo-electron microscopy single particle analysis. These problems have specific properties such as extremely high dimensionality and poor signal to noise ratio in the collected data. Their numerical solution is therefore highly challenging and strong regularization needs to be applied. We give a brief overview of the problem formulation and its discretization. The key parts of the solution of such problems including their implementation will be summarized and illustrated on numerical experiments with realistic data.

Inexact preconditioning for iterative methods

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Abstract

Preconditioning is almost always necessary when using Krylov subspace methods to solve linear systems. Designing a preconditioner for a problem is a complex task: one must balance the convergence rate of the algorithm with the cost of applying the preconditioner. Therefore there is a large interest in designing preconditioners that are reasonably efficient, but also inexpensive to apply. This project involves studying this area of inexact preconditioning, where inexactness may be introduced intentionally or may be due to use of finite (especially low) precision arithmetic. We present initial work towards this project involving the use of Incomplete LU factorization with-in a GMRES-IR based iterative refinement scheme, called GMRES-IR. We investigate the resulting cost per iteration and convergence rate compared to GMRES-IR with complete LU factorization.

Mixed precision numerical linear algebra

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Abstract

Low-precision arithmetic, in particular half-precision (16-bit) floating-point arithmetic, is now available in commercial hardware. It is anticipated that exascale machines, capable of 10^{18} floating point operations per second, will expose a range of hardware precisions through GPUs and other accelerators. Using lower precision can offer significant savings in computation and communication costs with proportional savings in energy. However, lower precision implies less accurate computations (e.g., machine epsilon for IEEE half precision is around 10^{-4}). A goal is therefore to develop mixed precision algorithms, which use low and high precision in different parts of the computation to achieve both performance and accuracy. The efficient use of such mixed-precision algorithms in numerical linear algebra can provide significant acceleration to scientific computing. We present initial work on this topic, which involves developing a three-stage mixed precision iterative refinement solver for linear systems. This solver combines existing mixed-precision approaches to balance performance and accuracy.

Numerical solution of degenerate parabolic equations: A priori analysis

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Abstract

The aim of this talk is to introduce the mathematical model commonly used in porous media flow problems, and to present a novel method for its numerical solution.

This model consists of Richard's equation coupled with appropriate boundary and initial conditions. The Richard's equation is a quasilinear parabolic partial differential equation which can degenerate to an elliptic equation or ordinary differential equation in real applications. It is challenging to develop and analyse a sufficiently accurate and efficient method for its numerical solution.

We consider the adaptive higher-order space-time discontinuous Galerkin (hp-STDG) finite element method proposed in [Dolejsi,Kuraz,Solin,2019]. The numerical experiments in this paper demonstrated that the proposed method is a very promising tool for solving such problems.

However, there are several issues that remain open. Namely, the existence of the approximate solution, convergence of the numerical scheme, and a priori and a posteriori error estimates need to be investigated. In this talk, we focus on a priori error estimate analysis, particularly possible issues related to its derivation and the strategy that we will follow in the future work.

Numerical solution of degenerate parabolic equations: A posteriori analysis

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Abstract

This presentation is a brief introduction to ongoing research about the a posteriori error estimate based on Richard's equation, as well as some physical and mathematical background. Richard's equation describes flows in porous media, and, since research about such the flows has been widely conducted, the numerical study of Richard's equation is considered significant in science and engineering. A posteriori error estimation is used to calculate the error from numerical results; therefore, it helps to understand how precise the numerical results can be.

The presentation consists of five sections. First, the applications of analyzing porous media flows are discussed with specific examples, performed theoretically and numerically. Secondly, several versions of Richard's equation are introduced with their derivation and associated physics. Next, the characteristics of the a posteriori error estimate are compactly explained. This estimate is then compared with the a priori error estimate to help understand both estimates. Lastly, a simple example is presented.

Exact Lanczos algorithm

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Abstract

In theory, the Lanczos algorithm generates an orthogonal basis of the corresponding Krylov subspace. However, in finite precision arithmetic, the orthogonality and linear independence of the computed Lanczos vectors is usually lost quickly. In this presentation the speaker will parametrize a class of matrices and starting vectors having a special nonzero structure that guarantees exact computations of the Lanczos algorithm whenever floating point arithmetic satisfying the IEEE 754 standard is used. Analogous results will be shown for a variant of the conjugate gradient method that produces almost exact results. Finally, we will discuss the usage of the obtained results in the analysis of theoretical as well as finite precision behaviour of the considered algorithms.

Theory and numerical solution of traffic flow models

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Abstract

Modelling of traffic flows will have an important role in the future. With a rising number of cars on the roads, we must optimize the traffic situation. That is the reason we started to study traffic flows. It is important to have working models which can help us to improve traffic flow. We can model real traffic situations and optimize e.g. the timing of traffic lights or local changes in the speed limit. The benefits of modelling and optimization of traffic flows are both ecological and economical.

There are two fundamental approaches: microscopic and macroscopic. The microscopic approach follows every single car. The macroscopic approach consider the traffic to be a continuum which flows through the roads. We describe a numerical technique for the solution of macroscopic traffic flow models on networks of roads. On individual roads, we consider the standard Lighthill-Whitham-Richards model which is discretized using the discontinuous Galerkin method along with suitable limiters. In order to solve traffic flows on networks, we construct suitable numerical fluxes at junctions based on preferences of the drivers. We present numerical experiments, including a junction with complicated traffic light patterns with multiple phases. Differences with the approach to numerical fluxes at junctions from Čanić et al., 2015, are discussed and demonstrated numerically on a simple network.

The work of L. Vacek is supported by the Charles University, project GA UK No. 1114119. The work of V. Kučera is supported by the Czech Science Foundation, project No. 20-01074S.

Multilevel methods with inexact solver on the coarsest level

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Abstract

Multilevel methods compute approximate solutions of a problem using its formulation on different levels. The approximate solutions are computed using smoothing on fine levels and solving on the coarsest level. The analysis of multilevel methods is typically carried out under the assumption that the problem on the coarsest level is solved exactly. This assumption is, however, not satisfied in practical computations due to the use of an iterative solver on the coarsest level, finite precision arithmetic, or both. In this talk, I will briefly present the results from my master thesis in which we studied convergence behavior of multilevel methods with inexact solver on the coarsest level. Further, I will describe our current research in which we focus on residual-based error estimates for the total error in multilevel frameworks. Our goal is to describe connections between estimates presented in literature and derive generalized results. In the end, I will touch on the topics I would like to study in the future such as the use of recycled Krylov methods as the solver on the coarsest level, stopping criteria for iterative solvers on the coarsest level, or effects of finite precision arithmetic.

Mathematical games of Sid Sackson

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Abstract

We will briefly introduce professional game inventor Sid Sackson and his work. We will focus on two games from his book 'A Gamut of Games' ([1]), namely 'Cutting Corners' and 'Hold That Line'. Both were mentioned in an inspiring paper 'Mathematical Treasures from Sid Sackson' ([2]) as examples of old games without known winning strategies. One year later a winning strategy for 'Hold That Line' was found and described in [3]. We will introduce main ideas of that strategy.

References

[1] S. Sackson, *A Gamut of Games*, Dover Publications, 1992.

[2] J. Henle, Mathematical treasures from Sid Sackson, *Math. Intelligencer* **41**:1 (2019), 71–77.

[3] J. Michalik, A winning strategy for Hold That Line, *Math. Intelligencer* **42**:4 (2020), 71–77.

Walking through a wall using 4D space

Mgr. Jakub Řada

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Abstract

4D space is complicated to visualise in 3D space. We can naturally imagine only the x, y, z axes. In our real world, we lack the w-axis, which would be perpendicular to all three axes mentioned above. For this reason, we can never imagine the whole 4D space. In this contribution, we would like to illustrate different methods of visualising a 4D space. We will demonstrate it through various projections or perceive it only through its intersections with a plane or space. We will invite 3D printing to help and show the 3D printed hypercubes on a 3D printer. Furthermore, we will mention why it is good to watch out for 4D creature, which we will probably never recognize in any case.

Stochastic methods in microstructure analysis

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Abstract

Stochastic models are frequently used in the field of modelling polycrystalline materials. The overall structure itself is usually modelled by tessellations. Our focus is mainly on understanding advanced grain characteristics such as stress within grains, grain orientations and normals of grain boundaries. These characteristics play an essential role in the mechanical properties of the material.

We consider a random marked tessellation in which the marks are crystal lattice orientations, volumes, grain-wise stress etc. The task is to develop some statistical methods for better understanding the inner dependencies, meaning both the spatial dependencies within the set of marks and the dependencies between the marks and the underlying unmarked tessellation. We have already constructed a non-parametric statistical test to decide whether the orientations are independently assigned to the grains of the underlying tessellation.

Generalisations of Lloyd-Sudbury dualities for IPS

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Abstract

In the study of interacting particle systems (IPS) duality is an important tool used to prove, for example, convergence to an invariant distribution or the existence of clusters. While the two most used types of dualities are additive and cancellative dualities [1], Lloyd and Subdury [2, 3, 4], have defined more general duality functions, that are able "interpolate" between additive and cancellative dualities. To gain a better understanding why this approach is very successful in practice, we leave the well-known case of IPS taking values in $\{0,1\}^\Lambda$, where Λ is some lattice, and generalise the ideas of Lloyd and Sudbury to IPS taking values in $\{0,1,\ldots,n-1\}^\Lambda$ for general $n \geq 2$. In this talk I will focus on the case $n = 3$, discuss strategies how to define promising duality functions and test them on the voter model as an example.

References

[1] D. Griffeath, *Additive and Cancellative Interacting Particle Systems*, Lecture Notes in Math. 724, Springer, Berlin, 1979.

[2] A. Sudbury, P. Lloyd, Quantum operators in classical probability theory. II: The concept of duality in interacting particle systems, *Ann. Probab.* **23**:4 (1995), 1816–1830.

[3] A. Sudbury, P. Lloyd, Quantum operators in classical probability theory. IV: Quasi-duality and thinnings of interacting particle systems, *Ann. Probab.* **25**:1 (1997), 96–114.

[4] A. Sudbury, Dual families of interacting particle systems on graphs, *J. Theor. Probab.* **13**:3 (2000), 695–716.

Stabilization of dynamical systems by noise with jumps

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Abstract

The aim of this talk is to present recent work on stabilization of finite-dimensional stochastic differential equations by Lévy noise. The results extend [1] and [2] in some directions. Here we put emphasis on geometric interpretation and investigate conditions under which both small and large jumps of the driving process stabilize the system.

References

[1] D. Applebaum, M. Siakalli, Asymptotic stability of stochastic differential equations driven by Lévy noise, *J. Appl. Probab.* **46**:4 (2009), 1116–1129.

[2] D. Applebaum, M. Siakalli, Stochastic stabilization of dynamical systems using Lévy noise, *Stoch. Dyn.* **10**:4 (2010), 509–527.

Stochastic approaches to reserving in non-life insurance

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Abstract

Stochastics can be used in various fields such as e.g., finance and insurance. This contribution will be focused on the calculation of technical reserves, which is one of the fundamental activities of insurance companies. The available data, which consists of incurred claims, are usually considered in the form of run-off triangles. Prediction of unknown values, which are the basis for the calculation of the reserves, can be constructed not only using simple approaches such as the chain-ladder, but also using advanced methods. State-space modeling is one of the possible options. Moreover, one can also generalize these methods to a multidimensional case, in order to deal with dependent lines of business of an insurance company at once. Since not only the reserves, but also their quantiles or even the whole distributions of the reserves, are important, this is another area of interest. The presentation will also include a numerical demonstration for better understanding the principle of reserving.

Mathematical models describing the inelastic response of solids

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Abstract

We review several phenomenological models describing the inelastic response of solids. Special attention is paid to the deformation of filled rubbers that exhibit so-called Mullins effect. When a rubber material is loaded from its virgin state, unloaded and then repeatedly loaded and unloaded, the stress required on reloading is less that on the previous loading. After introducing the models that have been so far mostly investigated in purely mechanical setting, we speculate on possible extensions to the full thermodynamical setting.

Fluid-structure interaction: non-standard interface conditions

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Abstract

Although fluid-structure interaction is a widely discussed topic, the interface conditions between interacting materials do not attract such attention. However, besides classical no-slip and full-slip boundary conditions in the rigid domains, there exist other less standard boundary conditions. In this presentation, we show how this generalization can be formulate. Finally, we present the numerical solution to fluid-structure interaction using the arbitrary Lagrangian– Eulerian method with these less standard interface conditions.

Variational data assimilation in computational haemodynamics

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Abstract

In this talk, we present some aspects of modelling of blood flow in the descending aorta. A motivation for our work is that having a good mathematical model for blood flow problems could help to improve predictive possibilities regarding various aortic diseases.

We show some types of magnetic resonance images which can be obtained for a given patient and how to use them in the simulation process. Next, we explain what is the variational assimilation and how it can help us incorporate magnetic resonance velocity images to our blood flow model. Then, we present some practical methods to solve a variational assimilation problem. We also present some experiments to demonstrate the approach in practice.

Uniqueness and regularity results for fluid-structure interaction problems, and related subjects

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Abstract

The fluid-structure interaction (FSI) systems are multi-physics systems that include a fluid and solid component. They are everyday phenomena with a wide range of applications. The simplest model for the structure is a rigid body. We study a nonlinear moving boundary fluid-structure interaction problem where the fluid flow is governed by 3D Navier-Stokes equations, and the structure is a rigid body described by a system of ordinary differential equations called Euler equations for the rigid body. We prove a generalization of the well-known weak-strong uniqueness and regularity result for the Navier-Stokes equations to the fluid-rigid body system. We also deal with related subjects such as the existence and regularity of pressure. This is a joint work with Boris Muha and Šárka Nečasová.

Non-equilibrium thermodynamics of hyperbolic systems

Mgr. Martin Sýkora

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Abstract

One way to describe the world around us is Hamiltonian mechanics. It can describe classical problems as easy as a single particle in one dimensional motion, such as an apple falling from a tree, as well as overwhelmingly complex systems of mixtures of interacting continua in complex geometries. In this talk, we briefly summarise the basic ideas of Hamiltonian mechanics and then apply them to binary mixtures. We compare the results with SHTC model that boasts the property of being a set of first order hyperbolic equations, thus having characteristics that may help construct numerical solutions.