Algebra II

Exercises for week 8

Problem 1. Let $F \leq E$ be a field extension. What is the relationship between the characteristics of E and F?

Problem 2. Prove on your own that if $E \leq F \leq K$ are fields then

$$[K : E] = [K : F][F : E].$$

Problem 3. Prove (if you don't know this already) that if R is a domain and I a maximal ideal of R then R/I is a field. Why is this relevant to field extensions?

Problem 4. Find the splitting field of $x^4 - 5x^2 + 6$ over $\mathbb{Q}$.

Problem 5. Find the minimal polynomial of $\sqrt{3} + \sqrt{5}$ over $\mathbb{Q}$.

Problem 6. Let p, q be distinct primes. Prove that $1, \sqrt{p}, \sqrt{q}, \sqrt{pq}$ are linearly independent over $\mathbb{Q}$.

Problem 7. Construct an extension of $\mathbb{Z}_3$ of infinite degree.

Problem 8. Let $F \leq E$ be fields. An element a of a field E is *algebraic* over the field F if there is a nonzero polynomial $p \in F[x]$ with a as its root. The extension $F \leq E$ is algebraic if all elements of E are algebraic over F. If $F \leq E$ and $E \leq K$ are algebraic extensions, is $F \leq K$ also algebraic?