Algebra II

Exercises for week 6

Problem 1. Find $a \in \mathbb{C}$ such that

$$2x^6 - x^5 - 11x^4 - x^3 + ax^2 + 2ax + 8$$

has 2 as its root.

Problem 2. Find a polynomial in $\mathbb{C}[x]$ with roots 1/2, i, 2-i.

Problem 3. Find all rational roots of the polynomial

$$12x^6 + 8x^5 - 85x^4 + 15x^3 + 55x^2 + x - 6.$$

Problem 4. Describe all irreducible polynomials in $\mathbb{R}[x]$.

Problem 5. Find all at least double complex roots of the polynomial

$$16x^4 - 88x^3 + 108x^2 - 50x + 8.$$

Problem 6 (needs a trick). Find a polynomial in $\mathbb{Z}[x]$ with a root $\cos(\pi/7)$.