Algebra II

Exercises for week 3

Problém 1. Calculate the greatest common divisor of $p(x) = x^4+2x^3+x^2+2x$ and $q(x) = 2x^5+x^4+x+2$ in $\mathbb{Z}_3[x]$ and express this divisor as $p(x)r(x)+q(x)s(x)$ (Bézout's equality).

Problém 2. Factorize the polynomial $x^4 - x^2 - 2$ in

a) $\mathbb{C}[x]$

b) $\mathbb{R}[x]$

c) $\mathbb{Q}[x]$

d) $\mathbb{Z}_5[x]$

e) $\mathbb{Z}_3[x]$

Hint: If $p(\alpha) = 0$ for α member of a field T then $x - \alpha | p(x)$ over T.

Problém 3. Find a non-principal ideal in $\mathbb{Z}[x]$.

Problém 4 (from Allenby's book "Rings, Fields and Groups")**.** What is wrong with the following short proof that $\mathbb{Z}[x]$ is a UFD?

$\mathbb{Q}[x]$ is Euclidean and hence UDF. The domain $\mathbb{Z}[x]$ is a subring of $\mathbb{Q}[x]$ and hence has the same unique factorization as $\mathbb{Q}[x]$ has.