

## Exercises for week 1

**Problém 1.** Prove that for all  $n \in \mathbb{Z}$  we have  $7|n^7 - n$ .

**Problém 2.** Prove that the ring  $\mathbb{Z}_2 \times \mathbb{Z}_2$  (with operations defined component-wise) is not a domain.

**Problém 3.** Prove that the ring  $\mathbb{Z}[\sqrt{5}]$  is a domain.

**Problém 4.** In  $\mathbb{Z}[\sqrt{5}]$ , find at least three divisors of 4 that are (pairwise) not associated.

**Problém 5.** Show that in  $\mathbb{Z}[\sqrt{5}]$  there exist irreducible elements that are not prime.