AG nerovnost: $\forall n \in \mathbb{N}, \forall a_1, \dots, a_n \geq 0: \quad \sqrt[n]{a_1 \cdots a_n} \leq \frac{1}{n}(a_1 + \dots + a_n)$

Indukcí: $n = 1$: $\quad a_1 = a_1$ ✓

$n \to n+1$ $\quad$ Fix $a_1, \dots, a_n, a_{n+1} \geq 0$

Zjednodušení: bůno $a_1, \dots, a_{n+1} > 0$

bůno $a_1 \cdot \ldots \cdot a_{n+1} = 1 \longrightarrow$ jinak přejdu k $b_i = \dfrac{a_i}{\sqrt[1+n]{a_1 \cdots a_{n+1}}}$

AG pro b

$$\left(\sqrt[1+n]{b_1 \cdots b_{n+1}} = 1 \leq \frac{1}{n}(b_1 + \dots + b_{n+1}) = \frac{1}{\sqrt[1+n]{a_1 \cdots a_{n+1}}} \, \frac{1}{n}(a_1 + \dots + a_{n+1}) \right)$$

bůno $a_{n+1} > 1, \; a_n < 1$

Chceme ukázat: $1 \leq \frac{1}{n+1}(a_1 + \dots + a_{n+1})$

$$\overbrace{\frac{a_1}{\sqrt[n+1]{a_1 \cdots a_{n+1}}} \cdot \frac{a_n}{\sqrt[n+1]{a_1 \cdots a_{n+1}}} \cdots \frac{a_{n+1}}{\sqrt[n+1]{a_1 \cdots a_{n+1}}}}^{n+1} = \frac{a_1 \cdots a_{n+1}}{a_1 \cdots a_{n+1}}$$

Víme: $c_1 = a_1, \; c_2 = a_2 \; \dots \quad c_{n-1} = a_{n-1}, \; c_n = a_n a_{n+1}$

$c_1 \cdot \ldots \cdot c_n = 1; \quad 1 \leq \frac{1}{n}(c_1 + \dots + c_n)$

$n \leq a_1 + a_2 + \dots + a_{n-1} + a_n a_{n+1}$

$n+1 \leq a_1 + a_2 + \dots + a_{n+1} +$

$+ a_n a_{n+1} - a_n - a_{n+1} + 1$

$a_n(a_{n+1} - 1) - (a_{n+1} - 1)$

$= (a_n - 1)(a_{n+1} - 1) \leq 0$