I/6 (a) Najděte reálnou a imaginární část $\sin(2+i)$

Postup č. 1: $\sin(2+i) \overset{\text{definice}}{=} \dfrac{\exp(i(2+i)) - \exp(-i(2+i))}{2i} =$

$= \dfrac{\exp(-1+2i) - \exp(1-2i)}{2i} \overset{(E3),\ \frac{1}{i}=-i}{=} -\frac{i}{2}\left(\exp(-1)\exp(2i) - \exp(1)\exp(-2i)\right) =$

Věta II.5(4)

$\overset{}{=} -\frac{i}{2}\left(\exp(-1)(\cos 2 + i\sin 2) - \exp(1)(\cos(-2) + i\sin(-2))\right) =$

$= \frac{1}{2}\exp(-1)(-i\cos 2 + \sin 2) + \frac{1}{2}\exp(1)(i\cos(2) - \sin(-2)) =$

$\cos(-2) = \cos(2),\ \sin(-2) = -\sin 2$

$= \underbrace{\frac{1}{2}\sin 2 \cdot (\exp(-1) + \exp(1))}_{\text{reálná část}} + i\cdot\underbrace{\frac{1}{2}\cos(2)\cdot(-\exp(-1) + \exp(1))}_{\text{imaginární část}}$

Postup č. 2: $\sin(2+i) \overset{\text{Věta II.5(7)}}{=} \sin 2\cdot\cos i + \cos 2\cdot\sin i =$

Věta II.5(4)

$= \sin 2\cdot\cosh(-1) + \cos 2\cdot(-i)\cdot\sinh(-1) =$

cosh sudá, sinh lichá

$= \underbrace{\sin 2\cdot\cosh 1}_{\text{reálná část}} + i\cdot\underbrace{\cos 2\cdot\sinh 1}_{\text{imaginární část}}$

I/7 b) $\sin z - \cos z = i$ (hledáme všechna řešení v $\mathbb{C}$)

$\dfrac{\exp(iz) - \exp(-iz)}{2i} - \dfrac{\exp(iz) + \exp(-iz)}{2} = i \qquad / \cdot 2i$

$\exp(iz) - \exp(-iz) - i\exp(iz) - i\exp(-iz) = -2$

$(1-i)\exp(iz) - (1+i)\exp(-iz) + 2 = 0 \qquad / \cdot \exp(iz)$

$(1-i)\exp(2iz) + 2\exp(iz) - (1+i) = 0$

Označme $u = \exp(iz)$, dostaneme rovnici

$(1-i)u^2 + 2u - (1+i) = 0$

Diskriminant $D = 4 + 4(1+i)(1-i) = 4 + 8 = 12$

kořeny $\dfrac{-2 \pm \sqrt{12}}{2(1-i)} = \dfrac{-1 \pm \sqrt{3}}{1-i} = \dfrac{-1 \pm \sqrt{3}}{2}(1+i)$

$\cdot\quad e^{iz} = \frac{-1+\sqrt{3}}{2}(1+i) = \frac{\sqrt{3}-1}{\sqrt{2}}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$$\left[\ \left|\frac{\sqrt{3}-1}{2}(-1-i)\right| = \frac{\sqrt{3}-1}{2}\cdot\sqrt{2} = \frac{\sqrt{3}-1}{\sqrt{2}}\ \right]$$

$$iz = \ln\frac{\sqrt{3}-1}{\sqrt{2}} + i\left(\frac{\pi}{4} + 2k\pi\right),\ k\in\mathbb{Z}$$

$$z = \frac{\pi}{4} + 2k\pi - i\ln\frac{\sqrt{3}-1}{\sqrt{2}},\ k\in\mathbb{Z}$$

$\cdot\quad e^{iz} = \frac{-1-\sqrt{3}}{2}(1+i) = \frac{\sqrt{3}+1}{\sqrt{2}}\left(\cos\left(-\frac{3}{4}\pi\right) + i\left(-\frac{3}{4}\pi\right)\right)$

$$iz = \ln\frac{\sqrt{3}+1}{\sqrt{2}} + i\left(-\frac{3}{4}\pi + 2k\pi\right),\ k\in\mathbb{Z}$$

$$z = -\frac{3}{4}\pi + 2k\pi - i\ln\frac{\sqrt{3}+1}{\sqrt{2}},\ k\in\mathbb{Z}$$

všechna řešení