VII. Compute the following determinants

1. $\begin{vmatrix} 1 & -1 & 0 & -3 \\ 7 & -2 & 2 & -10 \\ 7 & -1 & 1 & -9 \\ 2 & 0 & -2 & -4 \\ 6 & -1 & 2 & -7 \end{vmatrix}$ **2.** $\begin{vmatrix} 1 & 2 & -1 \\ 2 & 3 & 0 \\ 0 & -1 & 1 \end{vmatrix}$ **3.** $\begin{vmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 2 & 1 & 3 \end{vmatrix}$ **4.** $\begin{vmatrix} 1 & 2 & -3 & 1 \\ 2 & 3 & -1 & 2 \\ 7 & -1 & 4 & 3 \\ 1 & 1 & -2 & -1 \end{vmatrix}$

5. $\begin{vmatrix} 1 & 2 & -1 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 0 \end{vmatrix}$ **6.** $\begin{vmatrix} 1 & 2 & 1 & 3 \\ 2 & 4 & 5 & 1 \\ 3 & 6 & 7 & 5 \\ 4 & 8 & 3 & 7 \end{vmatrix}$ **7.** $\begin{vmatrix} 246 & 427 & 327 \\ 1014 & 543 & 443 \\ -342 & 721 & 621 \end{vmatrix}$

8. Determine the determinant of the matrix made:
a) from the matrix from problem 2 by re-ordering rows in the order 2,3,1;
b) by multiplying the matrix from problem 3 by -1;
c) by re-ordering columns in the matrix from problem 4 in the order 4,2,1,3;
d) by multiplying the matrix form problem 7 by $1/100$;
e) by multiplying the matrices from problems 4 a 5;
f) as $A^T AB$, where A is the matrix from problem 1 and B is the matrix from problem 6;
g)* as AA^T, where A is the matrix from problem 1.

Find all the solutions of the following systems of linear equations

9. $\begin{array}{rl} x+2y-z &= 1 \\ 2x+3y \quad &= 1 \\ -y+z &= 1 \end{array}$ **10.** $\begin{array}{rl} x \quad -z &= -2 \\ -x+y \quad &= 1 \\ 2x+y+3z &= 13 \end{array}$ **11.** $\begin{array}{rl} x_1+2x_2-3x_3+x_4 &= -5 \\ 2x_1+3x_2-x_3+2x_4 &= 0 \\ 7x_1-x_2+4x_3-3x_4 &= 15 \\ x_1+x_2-2x_3-x_4 &= -3 \end{array}$

12. $\begin{array}{rl} x_1+2x_2-x_3+x_4 &= 2 \\ x_1 \quad -x_4 &= -1 \\ x_2+x_3 \quad &= 0 \\ x_1+2x_2 \quad &= -1 \end{array}$ **13.** $\begin{array}{rl} x_1+2x_2+2x_3+3x_4 &= 5 \\ 6x_1+15x_2+12x_3+25x_4 &= 42 \\ 2x_1+5x_2+4x_3+8x_4 &= 14 \\ x_1-x_2+2x_3-4x_4 &= -7 \end{array}$

14. For which vectors on the right-hand side does the system with the same matrix as the system in the previous problem have a solution?

Answers and hints. **1.** Determinant does not exist, it is not a square matrix. **2.** 1 **3.** 6 **4.** -84 **5.** 1 **6.** 0 **7.** -29400000 **8.** a) 1; b) -6; c) -84; d) -29.4; e) -84; f) 0 (because $\det B = 0$); g) 0 (one can proceed as follows: check that $h(A) < 5$, deduce (using, for example, the theorem on matrix multiplication and transformation) that $h(AA^T) < 5$, so $\det(AA^T) = 0$). **9.** $x = 5$, $y = -3$, $z = -2$ **10.** $x = 1$, $y = 2$, $z = 3$ **11.** $(1, 0, 2, 0)$ **12.** $(5, -3, 3, 6)$ **13.** infinitely many solutions of the form $(-3 - 2t, 4, t, 0)$, $t \in \mathbb{R}$ **14.** for those vectors (a, b, c, d), which satisfy $7a = b + d$.