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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% LIST OF THEOREMS FOR THE EXAM %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Explanation:
% the number at the end of line = the number of the theorem in the lecture notes
% the sign before the number:
%      *   these theorems are not explicitly included into
%           the exam questions. Anyway, the knowledge is assumed,
%           including the idea of a proof (in case the theorem
%           was proved during the lectures).
%
%      +   hard theorems included to exam questions
%
% no sign  easy theorems included to exam questions
%
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%
%%%% Chapter X
%
adding a unit to a Banach algebra % X.2
renorming a Banach algebra % * X.3
on multiplication of invertible elements % X.5
Neumann's series and properties of the group of invertible elements % X.6 and X.7
properties of the resolvent function % + X.8
nonemptiness of spectrum % + X.9
Gelfand-Mazur theorem % X.10
spectrum and polynomials % X.11 and X.12
formula for the spectral radius % + X.13
on spectrum with respect to a subalgebra % + X.15 and X.16
path integral with values in a Banach space % * X.17
holomorphic functional calculus % + X.18
properties of ideals and maximal ideals % X.19
factorization of a Banach algebra % X.21
properties of complex homomorphisms and  $\Delta(A)$  % X.22 and X.23
on maximal ideals and complex homomorphisms % IV.24
Gelfand transform and its properties % + IV.25
%
%%%% Chapter XI
%
basic properties of algebras with involution % XI.2
on spectral radius of a normal element in a  $C^*$ -algebra % XI.3 and XI.4
adding a unit to a  $C^*$ -algebra % * XI.5
automatic continuity of  $*$ -homomorphisms % XI.6
spectrum of a self-adjoint element % XI.8
Gelfand-Neimark theorem % XI.9
on one-to-one  $*$ -homomorphisms % * XI.11
spectrum with respect to a  $C^*$ -subalgebra % XI.12
Fuglede theorem % * XI.13
continuous functional calculus in unital  $C^*$ -algebras % + XI.14
continuous functional calculus in non-unital  $C^*$ -algebras % * XI.15
characterization of unitary elements and operators % XI.17 and XI.18
characterization of orthogonal projections % XI.20
characterization of partial isometries % XI.21
%
%%%% Chapter XII
%
on subsets of the spectrum % * XII.1
properties of the numerical radius % XII.3 including XII.2
structure of normal operators % XII.4

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spectrum of a self-adjoint operator % XII.5  
 polar decomposition % XII.6  
 on closed and closable operators % XII.10 including XII.8  
 on the inverse of a closed operator % XII.13  
 properties of the resolvent set, resolvent function and spectrum of an unbounded operator % XII.14  
 on operators with empty spectrum % XII.15  
 on the adjoint operator % \* XII.16  
 basic properties of adjoint operators % \* XII.17  
 on kernel and range % XII.18  
 on the graph of the adjoint operator % XII.19  
 adjoint operator and closedness % XII.21  
 properties of symmetric operators % XII.23  
 spectrum of a self-adjoint operator % XII.25, including XII.24  
 characterization of self-adjoint operators among symmetric ones % XII.26  
 properties of the Cayley transform % XII.27  
 on the range of the Cayley transform % XII.29 including XII.28  
 Cayley transform for self-adjoint operators % XII.30  
 %  
 %%% Chapter XIII  
 %  
 Lax-Milgram lemma % \* XIII.1  
 spectral measure of a normal operator % + XIII.2 including the construction  
 construction and properties of the measurable calculus % + XIII.4 including the construction  
 properties of an abstract spectral measure % XIII.6  
 integral of a bounded function with respect to a spectral measure % + XIII.8  
 integral of an unbounded function with respect to a spectral measure % + XIII.11  
 properties of  $\int f dE$  (for  $f$  possibly unbounded) % \* XIII.12  
 spectrum of  $\int f dE$  % XIII.13  
 spectral decomposition of a bounded normal operator % XIII.9 and XII.10  
 spectral decomposition of a self-adjoint operator % XIII.15, XIII.16 and XIII.17  
 on  $T^*T$  % \* XIII.19  
 on normal unbounded operators % \* XIII.20  
 spectral decomposition of an unbounded normal operator % \* XIII.21  
 diagonalization of a normal operator % \* XIII.24 and XIII.25  
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