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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% LIST OF THEOREMS FOR THE EXAM %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Explanation:
% the number at the end of line = the number of the theorem in the lecture notes
% the sign before the number:
%      *   these theorems are not explicitly included into
%           the exam questions. Anyway, the knowledge is assumed,
%           including the idea of a proof (in case the theorem
%           was proved during the lectures).
%      +   "difficult theorem" included with this status
%           to exam questions
% no sign  "easy theorem" included with this status
%           to exam questions
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
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%%%% Chapter V
%
description of the convex, balanced and absolutely convex hull % * V.2
generating the linear topology using a neighborhood base % + V.3 and V.4
characterization of continuous linear mappings % V.6
characterization of continuous linear functionals % V.7
relationship of continuous and bounded linear mappings % V.8
properties of HTVS of finite dimension % V.9 and V.10
characterization of finite-dimensional TVS % + V.11
metrizability of TVS % * V.12 and V.13
basic properties of Minkowski functionals % * V.15
on the Minkowski functional of a convex neighborhood of zero % + V.17 including V.16
generating topology using a family of seminorms % V.19 and V.20
metrizability of LCS % * V.22
characterization of normable TVS % V.23
continuity, boundedness and convergence in a topology generated by seminorms % V.24 (incl. V.21)
on absolutely convex hull of a compact set % V.27–V.29
Banach-Steinhaus theorem % V.30
open mapping theorem % + V.31
Hahn-Banach extension theorem and its applications % V.32–V.34
Hahn-Banach separation theorem and its applications % + V.35 and V.36
%
%%%% Chapter VI
%
basic properties of abstract weak topologies % * VI.1
dual to an abstract weak topology % VI.3 and VI.4
Mazur theorem % VI.6
boundedness and weak boundedness % + VI.8
weak topology on a subspace % * VI.9
polar calculus % * VI.11
bipolar theorem % VI.12
Goldstine theorem % VI.14
Banach-Alaoglu theorem % + VI.15 and VI.16
reflexivity and weak compactness % VI.17 and VI.18
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Chapter VII

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Pettis measurability theorem % + VII.3 including VII.2 (and its variants VII.5 and VII.4)

construction and properties of the Bochner integral % VII.7

characterization of Bochner integrability % VII.8

dominated convergence theorem for Bochner integral % VII.9

on the weak integral % VII.11

Bochner integral and a bounded operator % VII.12

definition and properties of Lebesgue-Bochner spaces % + VII.14

separability of Lebesgue-Bochner spaces % + VII.15

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Chapter VIII

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adding a unit to a Banach algebra % VIII.2

renorming a Banach algebra % \* VIII.3

on multiplication of invertible elements % VIII.5

Neumann's series and properties of the group of invertible elements % + VIII.6 and VIII.7

properties of the resolvent function % + VIII.8

nonemptiness of spectrum % + VIII.9

Gelfand-Mazur theorem % VIII.10

spectrum and polynomials % VIII.11

formula for the spectral radius % + VIII.12

on spectrum with respect to a subalgebra % + VIII.14 and VIII.15

path integral with values in a Banach space % VIII.16

holomorphic functional calculus % + VIII.17

properties of ideals and maximal ideals % VIII.18

factorization of a Banach algebra % VIII.20

properties of complex homomorphisms and  $\Delta(A)$  % + VIII.21 and VIII.22

on maximal ideals and complex homomorphisms % VIII.23

Gelfand transform and its properties % + VIII.24

basic properties of algebras with involution % VIII.26

on spectral radius of a normal element in a  $C^*$ -algebra % VIII.27 and VIII.28

adding a unit to a  $C^*$ -algebra % \* VIII.29

automatic continuity of  $*$ -homomorphisms % VIII.30

spectrum of a self-adjoint element % VIII.32

Gelfand-Neimark theorem % VIII.33

on one-to-one  $*$ -homomorphisms % \* VIII.35

spectrum with respect to a  $C^*$ -subalgebra % VIII.36

Fuglede theorem % \* VIII.37

continuous functional calculus in unital  $C^*$ -algebras % + VIII.38

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Chapter IX

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characterization of unitary operators % IX.1

on subsets of the spectrum % IX.2

properties of the numerical radius % + IX.4 (incl. IX.3)

structure of normal operators % + IX.5

characterization of orthogonal projections % IX.6

on spectrum and numerical range of a self-adjoint operator % IX.7

polar decomposition % IX.8

Hilbert Schmidt theorem % + IX.9

Schmidt representation of compact operators % IX.11

Lax-Milgram lemma % \* IX.12

construction and properties of the measurable calculus % + IX.15