Algorithms on Polynomials

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1 Theoretical part

1. Provide example of an integral domain which is:
   - Gaussian and Noetherian;
   - Gaussian and non-Noetherian;
   - non-Gaussian and Noetherian;
   - non-Gaussian and non-Noetherian.
2. A (commutative) ring R is *Artinian* iff any descending chain of ideals from R eventually stabilizes; in other words there is no infinite chain of ideals $I_1 \supset I_2 \supset I_3 \supset ...$ Provide example of an integral domain which is:
   - Artinian and Notherian;
   - non-Artinian and Noetherian;
   - non-Artinian and non-Noetherian.
3. Show that every Artinian integral domain is a field.
4. Is it true that:
   - every finite graph is terminating;
   - every terminating graph becomes a forest after forgetting its' orientation;
   - every (finite) non-oriented graph can be oriented to a terminating graph.
5. An element $nf(x)$ is called a *normal form* of an element x iff $nf(x)$ is a terminal and $x \xrightarrow{*} nf(x)$. Prove that a terminating graph G is convergent iff any element of G has a unique normal form. * Is termination condition really necessary for the equivalence to hold?
6. * Show that the problem of deciding whether a polynomial $p(\overline{x})$ belongs to a given ideal $I \leq \mathbb{Q}[\overline{x}]$ is NP-hard (and is co-NP-hard, as well).
7. ** Can you provide an example of a commutative ring which is Artinian and non-Noetherian?

2 Computational part

1. Study the first two sections of the sage tutorial.
   `https://doc.sagemath.org/html/en/tutorial/index.html`