GO – error estimates

By *hp*-adaptive discontinuous Galerkin method, solve the problem

$$-\nabla \cdot \mathbb{A}\nabla u + \nabla \cdot (\boldsymbol{b}\, u) = 0 \qquad \text{in } \Omega := [0,4]\times[0,4] \setminus [0,2]\times[0,2], \tag{0.1}$$

$$\mathbb{A}\nabla u \cdot \boldsymbol{n} = g_N \qquad \text{on } \Gamma_N = \Gamma_1 \cup \Gamma_2, \tag{0.2}$$

$$u = u_D \qquad \text{on } \Gamma_D := \partial\Omega \setminus \Gamma_N, \tag{0.3}$$

where $\mathbb{A} = 10^{-3}\mathbb{I}$, $\boldsymbol{b} = (x_2, -x_1)$, and

- $u_D = 1$ on the edge $\{x_1 = 0\}$;
- $g_N = 0$ on $\Gamma_1 := \{\Gamma; x_1 = 4\}$ and $\Gamma_2 := \{\Gamma; x_2 = 0\}$;
- $u_D = 0$ is set elsewhere.

Compare "residual error estimates" with the "goal-oriented error estimates" for

$J_V(u) = \int_E u(x)\,\mathrm{d}x$, $E := (2.5, 3.5)\times(2.5, 3.5)$, $J_V(u) = 0.20314158 \pm 10^{-8}$,

$J_B(u) = \int_{G_B} \boldsymbol{b}\cdot\boldsymbol{n}\, u\,\mathrm{d}S$, $G_B := \Gamma_1$, $J_B(u) = 0.07408122 \pm 10^{-8}$,

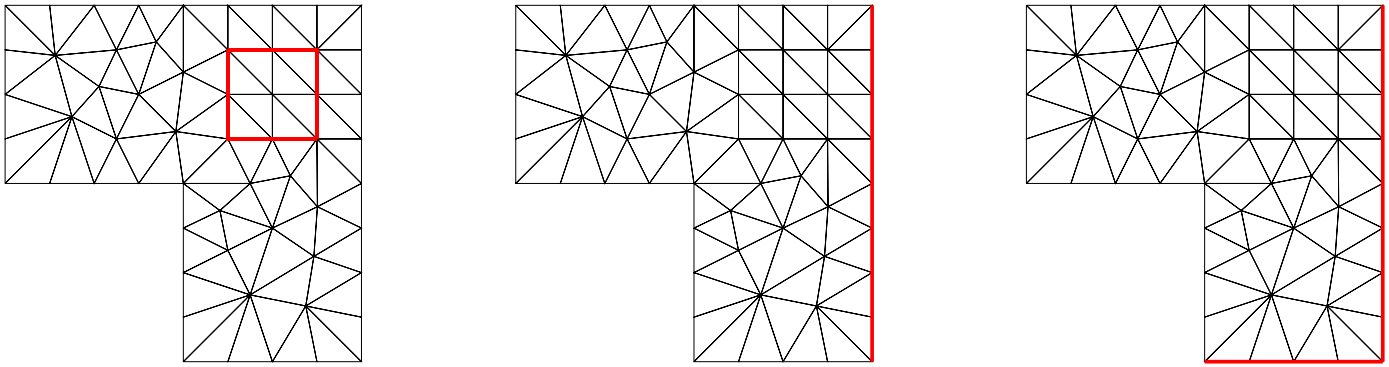
$J_D(u) = \int_{G_D} \boldsymbol{b}\cdot\boldsymbol{n}\, u\,\mathrm{d}S$, $G_D := \Gamma_1 \cup \Gamma_2$, $J_D(u) = 3.9670304 \pm 10^{-7}$.

Hint to solve:

- directory **BiCG** contains the following `.ini` files:
  - `carpioLshaped-interError.ini`
  - `carpioLshaped-JV.ini`
  - `carpioLshaped-JB.ini`
  - `carpioLshaped-JD.ini`
- running of the code: `../SRC_O/Adgfem carpioLshaped-interError.ini`
- visualize results by paraview

"domain of interest"

J_V J_B J_D



primal and dual solutions

primal dual J_V dual J_B dual J_D