

Homework 1 — Implicit RK

Numerical Solution for ODEs

Due date: November 20th, 2026

Support files for this homework can be found as a ZIP file on the webpage.

Exercise 1. Write a MATLAB implementation of *one* of the following Implicit Runge-Kutta methods:

RadauI2	RadauII2	Lobatto3	Lobatto3B	Lobatto3C
0 $\frac{1}{4}$ $-\frac{1}{4}$	$\frac{1}{3}$ $\frac{5}{12}$ $-\frac{1}{12}$	0 0 0 0	0 $\frac{1}{6}$ $-\frac{1}{6}$ 0	0 $\frac{1}{6}$ $-\frac{1}{3}$ $\frac{1}{6}$
$\frac{2}{3}$ $\frac{1}{4}$ $\frac{5}{12}$	1 $\frac{3}{4}$ $\frac{1}{4}$	$\frac{1}{2}$ $\frac{5}{24}$ $\frac{1}{3}$ $-\frac{1}{24}$	$\frac{1}{2}$ $\frac{1}{6}$ $\frac{1}{3}$ 0	$\frac{1}{2}$ $\frac{1}{6}$ $\frac{5}{12}$ $-\frac{1}{12}$
$\frac{1}{4}$ $\frac{3}{4}$	$\frac{3}{4}$ $\frac{1}{4}$	1 $\frac{1}{6}$ $\frac{2}{3}$ $\frac{1}{6}$	1 $\frac{1}{6}$ $\frac{5}{6}$ 0	1 $\frac{1}{6}$ $\frac{2}{3}$ $\frac{1}{6}$
		$\frac{1}{6}$ $\frac{2}{3}$ $\frac{1}{6}$	$\frac{1}{6}$ $\frac{2}{3}$ $\frac{1}{6}$	$\frac{1}{6}$ $\frac{2}{3}$ $\frac{1}{6}$

Initial templates for these methods (`radauI2.m`, `radauII2.m`, `lobatto3.m`, `lobatto3b.m` and `lobatto3C.m`) can be found in the support files.

Exercise 2. Test your script on the following problems from the support files:

1. `lin1p.m` for $t \in [0, 2]$, $x_0 = 2$, $\tau = 0.04$ and plot t versus the solution x :

```
x0=2.0; h=0.04;
figure;
[t,x]=feval(method, @lin1p,0,2, x0, h);
plot(t,x,'bo',t,x,'b');
```

2. `lin2.m` for $t \in [0, 0.1]$, $\mathbf{x}_0 = (2, 1)^\top$, $\tau = 0.001$ and plot t versus the solution x_1 :

```
figure;
x0 = [2;1]; h = 1e-3;
[t,x]=feval(method, @lin2, 0,.1, x0, h);
plot(t,x(:,1),'b');
```

3. `sat_ode.m` for $t \in [0, 6.19216933131963970674]$,

$$\mathbf{x}_0 = (1.2, 0, 0, -1.04935750983031990726)^\top,$$

$\tau = 0.001$ and x_1 versus x_2 :

```
figure
x0 = [1.2; 0; 0; -1.04935750983031990726]; h = 1e-3;
[t,x] = feval(method, @sat_ode, 0, 6.19216933131963970674, x0, h);
plot(x(:,1), x(:,2));
```

Save these plots as a PDF using **Save > Save As**. A function called `test_problems.m` is included in the support files, which performs the above operations when passed the name of the implicit Runge-Kutta method to run:

```
test_problems(@lobatto3);
```

Exercise 3. Estimate the order of the method by linear regression. See `conv_analysis.m` for a script to perform this, when called with the name of the implicit Runge-Kutta method:

```
conv_analysis(@lobatto3);
```

Submission

Submit the MATLAB script for the implemented method from *exercise 1*, the PDF files of the plots from *exercise 2*, and enter the order of the method deduced in *exercise 3* via the *Study Group Roster (Záznamník učitele)* in SIS before the deadline.