

Numerical Solution of ODEs

Exercise Class

2nd October 2026

Exercises

1. Read about the following ODE systems:

Population Growth http://en.wikipedia.org/wiki/Population_growth

Logistic http://en.wikipedia.org/wiki/Logistic_function

Pendulum http://en.wikipedia.org/wiki/Pendulum_%28mathematics%29

Harmonic Oscillator http://en.wikipedia.org/wiki/Harmonic_oscillator

2. Consider the logistic equation

$$\begin{aligned}x' &= (a - bx)x - c, \\x(t_0) &= x_0\end{aligned}$$

with non-negative constants a , b , and c . Solve using `ode23` with the right hand side function `logistic.m`, and plot the results, with the following different values:

constants: $a = b = 1$ and $c = 0$

time range: $t = [0, 3], [0, -1]$

initial condition: $x_0 = 1/2, 3/2, 1, -1/20$

3. Compare `ode23` and `ode15s` for the logistic equation with $a = b = 1$, $c = 1/5$, $t = [0, 100]$, $x_0 = 0.7233$. Compare also to the known exact solution:

$$u(t) = \frac{\sqrt{5}}{10} \tanh \left((t - t_0) \frac{\sqrt{5}}{10} + \operatorname{arctanh} \left((2x_0 - 1)\sqrt{5} \right) \right) + \frac{1}{2}.$$

4. Use `ode23` to solve the pendulum problem (`pendulum.m`)

$$\begin{aligned}x'' &= -k \sin(x) && \equiv && \begin{cases} x'_1 = x_2, \\ x'_2 = -k \sin(x) \end{cases} \\x(t_0) &= x_0\end{aligned}$$

with $k = 1$, $t = (0, 6\pi)$, and various initial conditions

$$x_0 = \begin{pmatrix} -1.5 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \end{pmatrix}, \begin{pmatrix} -\pi \\ 1 \end{pmatrix}.$$

5. Consider the harmonic oscillator

$$\begin{aligned}x'' + bx &= c \cos(\omega t), \\x(t_0) &= x_0\end{aligned}$$

- (a) Transform into a system of first-order ODEs
- (b) Attempt to derive an explicit solution. Consider the two cases $b \neq \omega^2$ and $b = \omega^2$ separately.
- (c) Use ode23 (oscillator.m) to solve with:
 - constants:** $a = 0, b = 9, c = 10, \omega = 2.5, 2.9, 3.1, 3, \sqrt{3}$
 - time range:** $t = [0, 50]$
 - initial condition:** $x_0 = (1, 0)^\top$