Adams-Bashfort ($m = 3$) As an explicit method Adams-Bashfort requires that $a_3 = 1$, $a_2 = -1$, $a_1 = a_0 = 0$, $b_3 = 0$, and by Theorem 3.2, with $p = 3$,

$$\left.\begin{array}{ll} \ell = 1: & a_1 + 2a_2 + 3a_3 = b_0 + b_1 + b_2, \\ \ell = 2: & a_1 + 4a_2 + 9a_3 = 2(b_1 + 2b_2), \\ \ell = 3: & a_1 + 8a_2 + 27a_3 = 3(b_1 + 4b_2); \end{array}\right\} \implies \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 4 \\ 0 & 3 & 12 \end{pmatrix} \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ 19 \end{pmatrix}$$

By Gaussian elimination

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 5 \\ 0 & 3 & 12 & 9 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 5 \\ 0 & 0 & 6 & 11.5 \end{array}\right] \to \left[\begin{array}{ccc|c} 12 & 12 & 12 & 12 \\ 0 & 12 & 24 & 30 \\ 0 & 0 & 12 & 23 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 12 & 12 & 0 & -11 \\ 0 & 12 & 0 & -16 \\ 0 & 0 & 12 & 23 \end{array}\right] \to \left[\begin{array}{ccc|c} 12 & 0 & 0 & 5 \\ 0 & 12 & 0 & -16 \\ 0 & 0 & 12 & 23 \end{array}\right].$$

Therefore,

$$b_2 = \frac{23}{11}, \quad b_1 = -\frac{4}{3}, \qquad b_0 = \frac{5}{12},$$

which gives

$$u_{j+3} = u_{j+2} + \tau \left(\frac{23}{12} f(t_{j+2}, u_{j+2}) - \frac{4}{3} f(t_{j+1}, u_{j+1}) + \frac{5}{12} f(t_j, u_j) \right).$$

Adams-Moulton ($m = 3$) As an implicit method Adams-Moulton requires that $a_3 = 1$, $a_2 = -1$, $a_1 = a_0 = 0$, $b_3 \neq 0$, and by Theorem 3.2, with $p = 4$,

$$\left.\begin{array}{ll} \ell = 1: & a_1 + 2a_2 + 3a_3 = b_0 + b_1 + b_2, \\ \ell = 2: & a_1 + 4a_2 + 9a_3 = 2(b_1 + 2b_2 + 3b_3), \\ \ell = 3: & a_1 + 8a_2 + 27a_3 = 3(b_1 + 4b_2 + 9b_3), \\ \ell = 4: & a_1 + 16a_2 + 81a_3 = 4(b_1 + 4b_2 + 27b_3); \end{array}\right\} \implies \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 3 & 12 & 27 \\ 0 & 4 & 32 & 108 \end{pmatrix} \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ 19 \\ 65 \end{pmatrix}$$

By Gaussian elimination

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 6 & 5 \\ 0 & 3 & 12 & 27 & 19 \\ 0 & 4 & 32 & 108 & 65 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 6 & 5 \\ 0 & 0 & 6 & 18 & 11.5 \\ 0 & 0 & 0 & 24 & 9 \end{array}\right] \to \left[\begin{array}{cccc|c} 24 & 24 & 24 & 24 & 24 \\ 0 & 24 & 48 & 72 & 60 \\ 0 & 0 & 24 & 72 & 46 \\ 0 & 0 & 0 & 24 & 9 \end{array}\right]$$

$$\to \left[\begin{array}{cccc|c} 24 & 24 & 24 & 0 & 15 \\ 0 & 24 & 48 & 0 & 33 \\ 0 & 0 & 24 & 0 & 19 \\ 0 & 0 & 0 & 24 & 9 \end{array}\right] \to \left[\begin{array}{cccc|c} 24 & 24 & 0 & 0 & -4 \\ 0 & 24 & 0 & 0 & -5 \\ 0 & 0 & 24 & 0 & 19 \\ 0 & 0 & 0 & 24 & 9 \end{array}\right] \to \left[\begin{array}{cccc|c} 24 & 0 & 0 & 0 & 1 \\ 0 & 24 & 0 & 0 & -5 \\ 0 & 0 & 24 & 0 & 19 \\ 0 & 0 & 0 & 24 & 9 \end{array}\right].$$

Therefore,

$$b_3 = \frac{3}{8}, \quad b_2 = \frac{19}{24}, \quad b_1 = -\frac{5}{24}, \qquad b_0 = \frac{1}{24},$$

which gives

$$u_{j+3} = u_{j+2} + \tau \left(\frac{3}{8} f(t_{j+3}, u_{j+3}) + \frac{19}{24} f(t_{j+2}, u_{j+2}) - \frac{5}{24} f(t_{j+1}, u_{j+1}) + \frac{1}{24} f(t_j, u_j) \right).$$