Numerical Solution of ODEs

Exercise Class

3rd October 2024

Exercises

1. Read about the following ODE systems:

   Population Growth `http://en.wikipedia.org/wiki/Population_growth`

   Logistic `http://en.wikipedia.org/wiki/Logistic_function`

   Pendulum `http://en.wikipedia.org/wiki/Pendulum_%28mathematics%29`

   Harmonic Oscillator `http://en.wikipedia.org/wiki/Harmonic_oscillator`

2. Consider the logistic equation

$$\begin{aligned} x' &= (a - bx)x - c, \\ x(t_0) &= x_0 \end{aligned}$$

   with non-negative constants a, b, and c. Solve using `ode23` with the right hand side function `logistic.m`, and plot the results, with the following different values:

   constants: $a = b = 1$ and $c = 0$

   time range: $t = [0, 3], [0, -1]$

   initial condition: $x_0 = {}^1/_2, {}^3/_2, 1, -{}^1/_{20}$

3. Compare `ode23` and `ode15s` for the logistic equation with $a = b = 1$, c=$^1/_5$, $t = [0, 100]$, $x_0 = 0.7233$. Compare also to the known exact solution:

$$u(t) = \frac{\sqrt{5}}{10} \tanh\left((t - t_0)\frac{\sqrt{5}}{10} + \operatorname{arctanh}\left((2x_0 - 1)\sqrt{5}\right)\right) + \frac{1}{2}.$$

4. Use `ode23` to solve the pendulum problem (`pendulum.m`)

$$x'' = -k\sin(x) \qquad \equiv \qquad \begin{cases} x_1' = x_2, \\ x_2' = -k\sin(x) \end{cases}$$

$$x(t_0) = x_0$$

   with $k = 1$, $t = (0, 6\pi)$, and various initial conditions

$$x_0 = \begin{pmatrix} -1.5 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \end{pmatrix}, \begin{pmatrix} -\pi \\ 1 \end{pmatrix}.$$

5. Consider the harmonic oscillator

$$\begin{aligned} x'' + bx &= c\cos(\omega t), \\ x(t_0) &= x_0 \end{aligned}$$

(a) Transform into a system of first-order ODEs

(b) Attempt to derive an explicit solution. Consider the two cases $b \neq \omega^2$ and $b = \omega^2$ separately.

(c) Use `ode23` (`oscillator.m`) to solve with:
constants: $a = 0$, $b = 9$, $c = 10$, $\omega = 2.5, 2.9, 3.1, 3, \sqrt{3}$
time range: $t = [0, 50]$
initial condition: $x_0 = (1, 0)^\top$