Optimization theory – practicals – sample test

Example 1. [3 p.] Is the following function quasiconvex?

$$f(x,y) = \frac{1}{xy} \text{ on } (0,\infty)^2.$$

Provide at least one definition of quasiconvex functions.

Example 2. [3 p.] Formulate the definition of the subgradient for the following function

$$h(x) = |x-1|$$

defined on $\mathbb{R}$. Does it hold $0 \in \partial h(1)$?

Example 3. [4 p.] Prove the inclusion between the set of improving directions for a differentiable function f and its (outer) approximation using the gradient, i.e. $F_f(x) \subseteq F'_{f,0}(x)$.

Example 4. [5 p.] Consider the problem

$$\begin{aligned} \min\ & -x \\ s.t.\ & x^2 + y^2 \le 1 \\ & (x-1)^3 - y \le 0. \end{aligned}$$

Using the KKT optimality conditions find all stationary points. Using the SOSC verify if some of the points corresponds to a (strict) local minimum.

Example 5. [5 p.] Using the KKT conditions find the closest point to (0,0) in the set defined by

$$M = \{x \in \mathbb{R}^2 :\ x_1 + x_2 \ge 4,\ 2x_1 + x_2 \ge 5\}.$$

Can several points (solutions) exist?

14 out of 20 points are necessary to pass