Universal Algebra 2 - Exercises 5

Exercise 5.1. Let $R \leq_{sd} \mathbb{A} \times \mathbb{B}$, where $\mathbb{A}, \mathbb{B}$ are finite, simple. Show that either R is the graph of a bijection $A \to B$ or R is linked.

Exercise 5.2. Let $\mathbb{A}, \mathbb{B}$ be finite idempotent, $C \subseteq \mathbb{A}$, $D \subseteq \mathbb{B}$, and $R \subseteq_{sd} C \times D$ linked. Prove that $\mathrm{Sg}_{\mathbb{A}\times\mathbb{B}}(R) \leq_{sd} \mathrm{Sg}_{\mathbb{A}}(C) \times \mathrm{Sg}_{\mathbb{B}}(D)$ is linked.

Exercise 5.3. Let A be finite and idempotent. Prove that $\forall a \in A, \{a\} \trianglelefteq \mathbb{A}$ iff $\mathrm{Clo}(\mathbb{A})$ contains a near-unanimity operation.

$$n(x, \ldots x, y, x, \ldots, x) \approx x$$

Exercise 5.4. Let $\mathbb{A}$ be finite Taylor, $\alpha, \beta \in \mathrm{Con}(\mathbb{A})$ such that $\alpha \wedge \beta = 0$ and $\alpha \vee \beta = 1$. Prove that $\alpha \circ \beta = 1$ $(= \beta \circ \alpha)$ or A contains a proper absorbing subalgebra. (Hint: look at A "$\leq$" $A/\alpha \times A/\beta$)

Exercise 5.5. Prove that $B \subseteq \mathbb{A}$ is projective iff $\forall n, R_n \leq \mathbb{A}^n$; where $R_n(x_1, \ldots, x_n)$ iff $B(x_1) \vee \cdots \vee B(x_n)$ (i.e., $R_n = A^n \setminus (A \setminus B)^n$).

Exercise 5.6. Assume that A is idempotent and no subalgebra of A has a proper 2-absorbing subalgebra. Prove that $\mathrm{Clo}(\mathbb{A})$ contains a cube operation.