Universal Algebra Exercises - Homework 5

Exercise 1. Let $L = (\{0, 1, 2\}, \wedge, \vee)$ be the three-element lattice. Find a monotone, idempotent operation that is not in $\mathrm{Clo}(L)$.

Exercise 2. Let A be a set and $R := \{(x, y, z) \in A^3 \mid x = y \text{ or } y = z\}$. Show that $\mathrm{Pol}(R)$ is the clone of all essentially unary operations, meaning that all operations are of the form $(x_1, \ldots, x_n) \mapsto f(x_i)$ for some $f : A \to A$.

Exercise 3. Recall that $\mathcal{C} = \mathrm{Pol}(\mathrm{Inv}(\mathcal{C}))$ if $\mathcal{C}$ is a clone on a finite set A. Show that this is not true if A is infinite (Hint: consider the clone generated by all bijections $A \to A$).