Mathematics and Interviews

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My interview

For us, markets are places for fair exchange and ways to make the world a better place, not just a mathematical money-making game.

Being passionate advocates of technology and free markets, we believe that when our software improves markets for everyone, we all benefit.

Job interview

Is it an exam or a goodness of fit?

Is it a one-way or two-way assessment?

The interviewer is your friend and will try to help you.

Job interview

What challenges do we meet during interviews?

We usually spend a lot of time solving a problem.

Yet, we have very limited time during interview.

How to assess one’s ability to solve problems?

There’s usually a central topic which seems to be a recurring theme.

Job interview

Problem-solving skill is Alpha and Omega in Qminers.

Trading is very competitive and very difficult area.

Closed research, google search (mostly) doesn’t work.

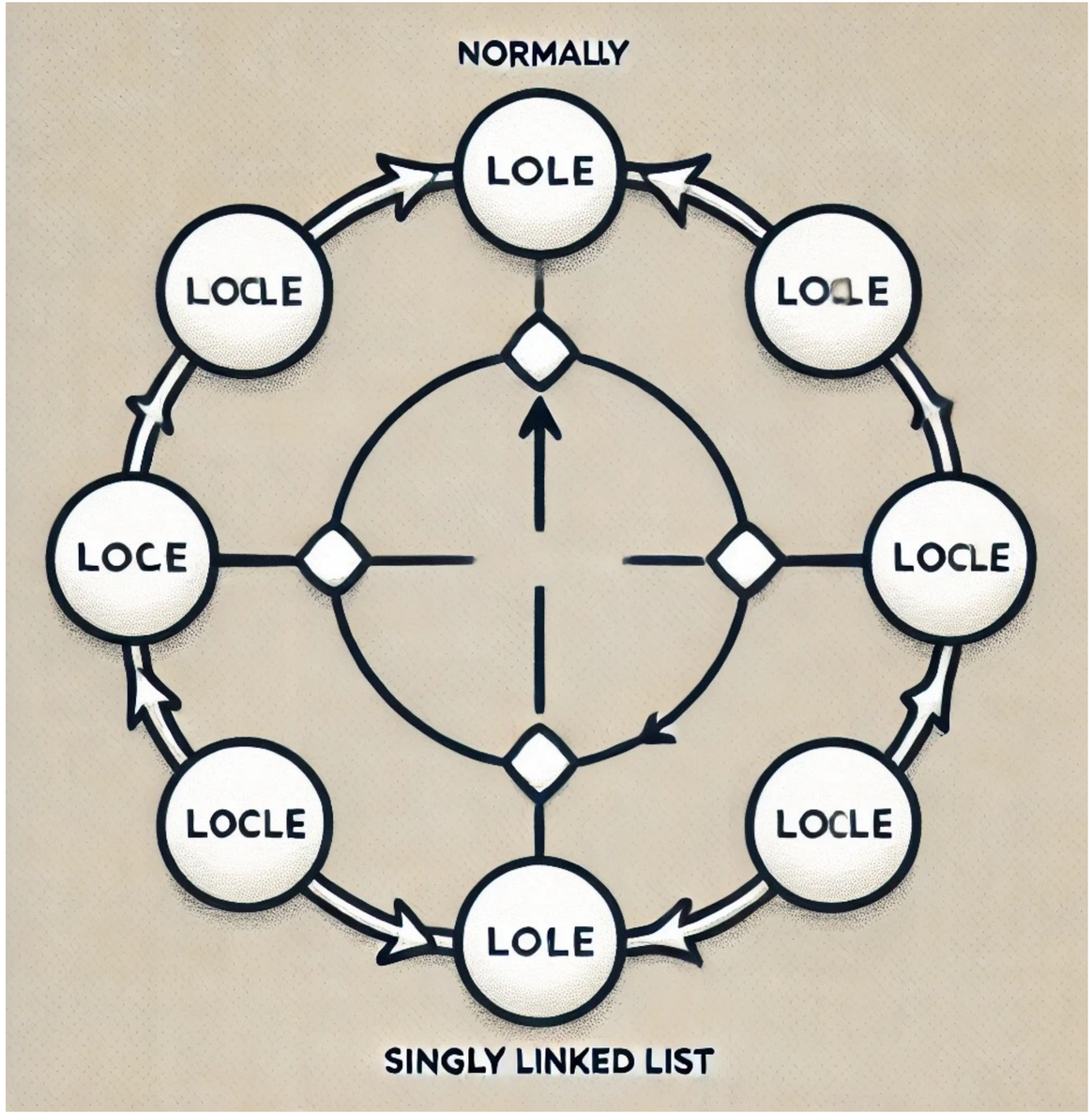
Be it an analyst or a developer, we all solve potentially hard problems.

Every analyst handles programming, developers are strong at math.

Cycle detection in singly linked list

Given a singly linked list, design an algorithm to detect if there is a cycle in the list.

Linear time, constant memory.



Cycle detection in singly linked list

The trick — slow and fast pointers.

A fresh graduate will likely know.

Senior developer hasn’t seen linked lists for years.

Outcome?

60 minutes of sweating and we only know what we knew beforehand.

Cycle detection in singly linked list

The least we can do — can you explain why the algorithm works?

Step 1: Cycle

$$k \equiv 2k \pmod{C}$$
$$0 \equiv k \pmod{C}$$

Step 2: General case

$$-t + t + k \equiv -t + 2t + 2k \pmod{C}$$
$$0 \equiv t + k \pmod{C}$$

Problem-solving strategies

Simplification/Reduction

Generalization

Abstraction

Problem-solving tools

Pen and paper

Sketching, drawing, scratching

Playing and exploration

Simple concepts, proofs and reasoning

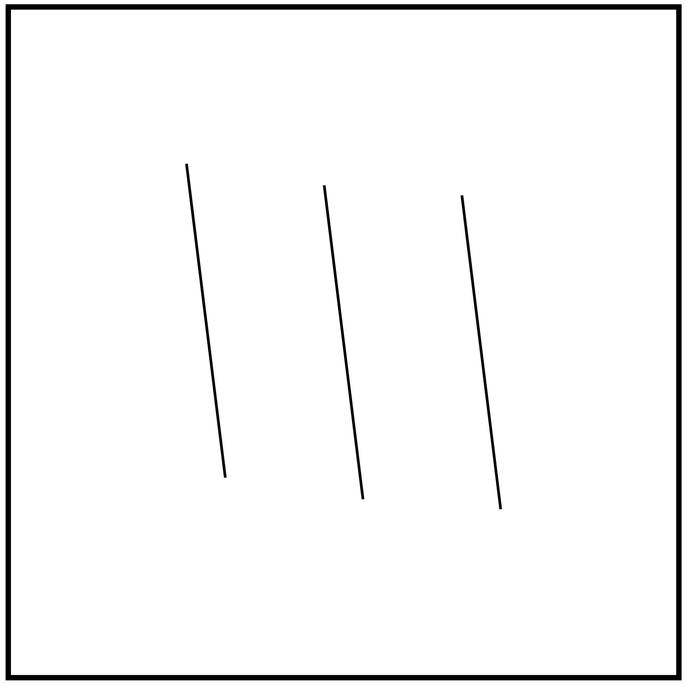
Listening and communication

Impossible square

Draw a square with 3 lines.

Impossible square

Draw a square with 3 lines.



Card counting trick

Alice and Bob use a standard deck of 52 cards to present a magical trick.

Alice randomly selects 5 cards and 4 of them hands over to Bob. Based on these 4 cards, Bob correctly guesses the 5th card.

What’s the trick?

Card counting trick

Indeed, there’s a clever solution, but discussion may be even more interesting.

- Combinatorics
- Codes and encodings
- Memory addressing
- Complexity and trade-offs

Card counting trick

The smart solution:

1. Index cards from 0 to 51.
2. Sort the selected cards.
3. Take sum of the 5 selected cards modulo 5 and hide card at this position.
4. Permute the remaining 4 cards to encode the hidden card number.

Bonus question:

What is the maximum size of card deck?

Pile of coins

There is a pile of 100 coins on the table and exactly 10 of them are heads up.

Your goal is to split the pile into two piles such that each pile contains same number of coins heads up.

You are blindfolded.

Pile of coins

What the actual are to solve the problem?

- Probability?
- Error-correction codes?
- Combinatorics?

Let me give you a hint…

Pile of coins

Your goal is **not** to

> Split the pile into two **equal** piles, such that each pile contains **5 coins** that are heads up.

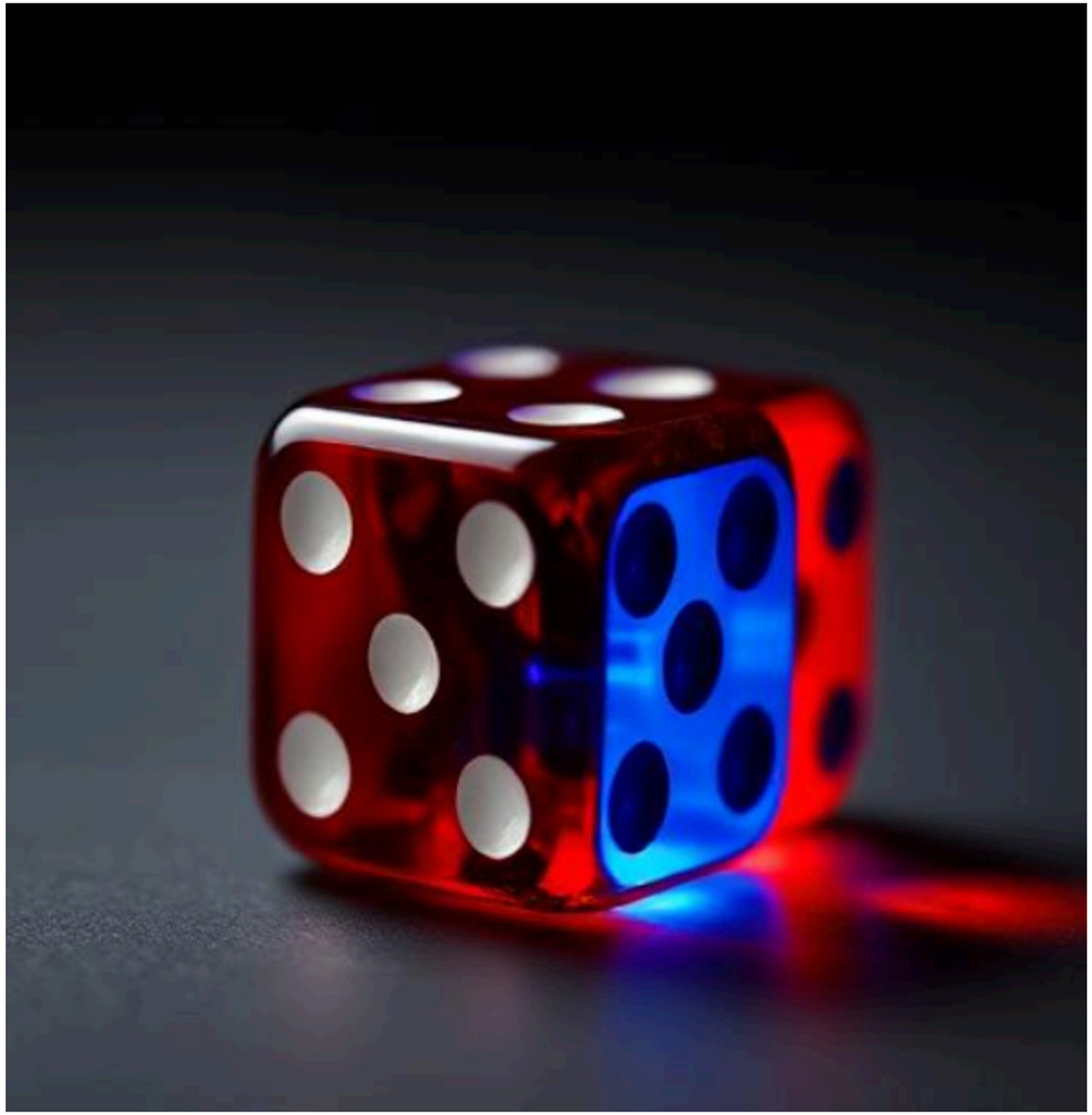
Just

> Split the pile into two piles, such that each pile contains same number of coins that are heads up.

Probability foundations

Let’s perform an infinite series of tosses of a fair coin. HHTHTTT… is such a sequence. Define sample space Ω to be a set of all such sequences.

What is the probability that a any sampled sequence starts by two tails?



Probability foundations

Probability is a function

$f : \Omega \to [0,1]$ satisfying positivity, normalization, additivity axioms.

Sample space is

$$\Omega = \{HHTH \ldots, TTHH \ldots, \ldots\}$$

Probability foundations

Create a r.v. $X : \Omega \to [0,1]$ s.t.

$$H \to 0, T \to 1$$
$$C_1 C_2 C_3 \ldots \to 0.D_1 D_2 D_3 \ldots_2$$

The probability that a sampled series starts with two tails is

$$\int_{0.1100\ldots}^{0.1111\ldots} dx = \int_{0.75}^{1.0} dx = \frac{1}{4}$$

Bonus question: Is X defined correctly? How to fix it?

Marbles in box

Box contains red and blue marbles. At each step, two marbles are removed from the box at random.

If the marbles are of the same color, red one is put back. If the marbles are of different colors, blue one is put back.

What will be the color of the last remaining marble?

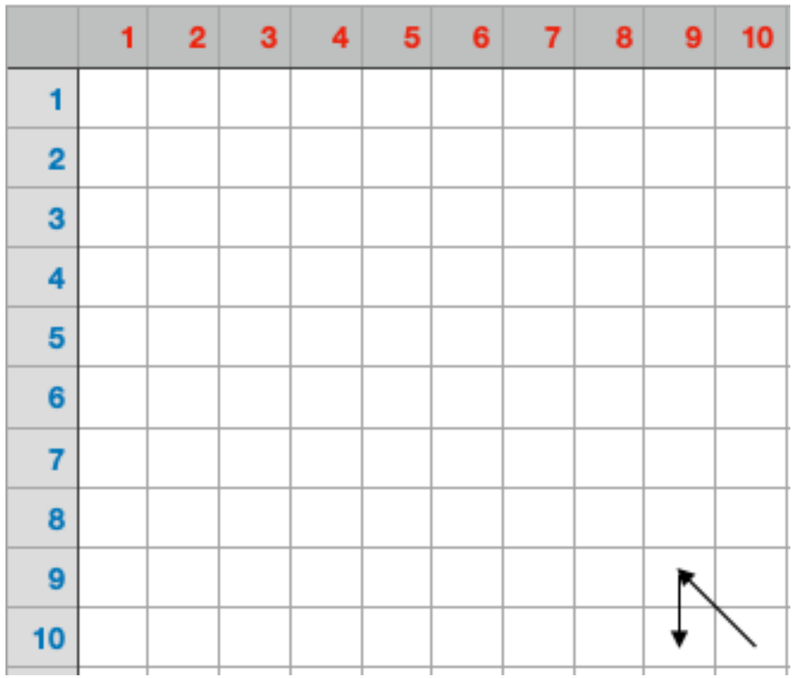
Marbles in box

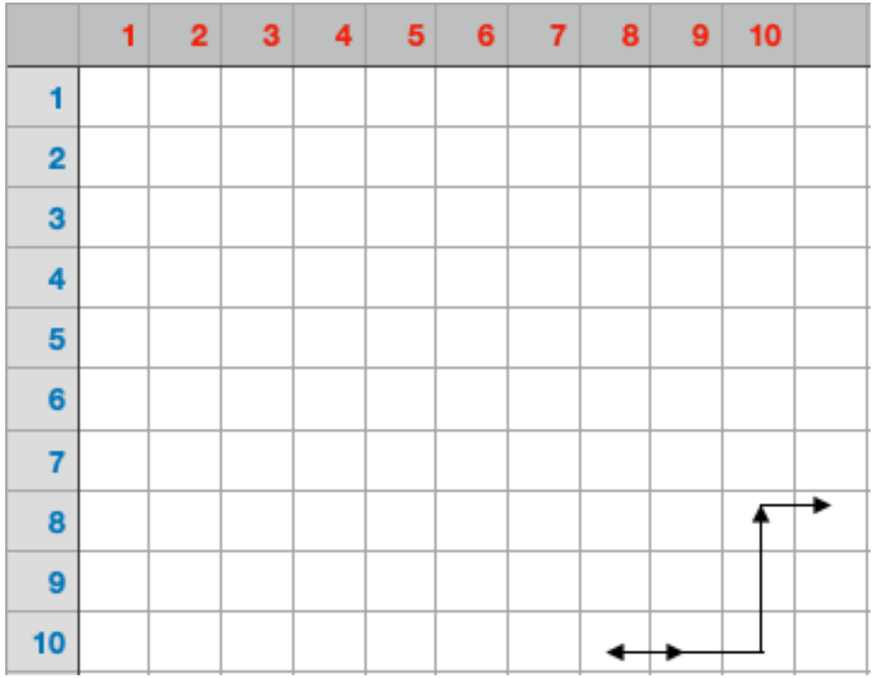
Are we interested in a formal solution?

W.r.t. interview it’s likely the least interesting part.

Yet, you should still be able to define rules and invariants.

Marbles in box





References

Mathematical Puzzles, Peter Winkler

Cracking the Coding Interview, G.L.McDowell

YT MindYourDecisions, Presh Talwalkar

YT 3Blue1Brown, Grant Sanderson

YT Veritasium

YT Mathologer, Burkard Polster

Takeaways

It’s all about the fit.

Simplify, simplify more, then generalize.

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