

AUTOMATA, LANGUAGES AND MONOIDS III

Simon's theorem; Piecewise-testable languages

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24th November 2014

Theorem

Let $L \subseteq A^$ be a regular language. Then L is piecewise testable if and only if its syntactic monoid is \mathcal{J} -trivial.*

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Proof.

Obvious. □

Definition

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\mathcal{C}_n is the submonoid of \mathcal{T}_n formed by *increasing* and *extensive* functions from $\{1, 2, \dots, n\}$ into itself, i.e. functions f such that:

- (a) $i \leq j \Rightarrow f(i) \leq f(j)$
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Proposition

For every integer $n > 0$, the monoid \mathcal{C}_n , is \mathcal{J} -trivial.

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Theorem

Let M be a finite monoid. The following conditions are equivalent:

- (1) M is \mathcal{J} -trivial,*
- (2) M divides \mathcal{C}_n for some n ,*
- (3) M divides \mathcal{R}_n for some n .*

Wake up!