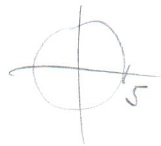
(3) $f = x^2+y^2+16x-12y$ $\qquad$ $M: \quad x^2+y^2 \leq 25$



- M is bounded, closed $\to$ compact
  f is continuous (polynomial) $\Big\}$ f attains extrema

- int M:

$$\frac{\partial f}{\partial x} = 2x+16 \qquad x = -8$$

$$\frac{\partial f}{\partial y} = 2y-12 \qquad y = 6$$

$[-8, 6] \notin M$

- ∂M:

$$g = x^2+y^2-25$$

$$\nabla g = (2x, 2y) \to [0,0] \notin M$$

- Lagr. mult.:

$$2x+16 + 2x\lambda = 0 \quad \to \quad 2x(1+\lambda) = -16$$

$$2y-12 + 2y\lambda = 0 \qquad x = \frac{-8}{1+\lambda}$$

$$x^2+y^2 = 25 \qquad (\lambda \neq -1) \text{ impossible}$$

$$2y(1+\lambda) = 12$$

$$y = \frac{6}{1+\lambda}$$

$$\frac{64}{(1+\lambda)^2} + \frac{36}{(1+\lambda)^2} = 25$$

$$64+36 = 25(1+\lambda)^2$$

$$4 = (1+\lambda)^2 \qquad \lambda_1 = 1 \to x = -4 \quad y = 3$$

$$\pm 2 = 1+\lambda \to \lambda_2 = -3 \to x = 4 \quad y = -3$$

- Suspect points

$f(-4,3) = 16+9-64-36 = -75$ glob min

$f(4,-3) = 16+9+64+36 = 125$ glob max

① $\begin{vmatrix} 1 & 0 & -1 & -4 \\ -2 & 3 & 0 & 1 \\ 2 & 0 & 3 & 4 \\ 1 & 1 & 2 & 3 \end{vmatrix} = (-1)^{2+2} \cdot 3 \begin{vmatrix} 1 & -1 & -4 \\ 2 & 3 & 4 \\ 1 & 2 & 3 \end{vmatrix}$

$+ (-1)^{4+2} \cdot 1 \cdot \begin{vmatrix} 1 & -1 & -4 \\ -2 & 0 & 1 \\ 2 & 3 & 4 \end{vmatrix}$

$= 3\left(9 - 4 - 16 - (-12 + 8 - 6)\right) + \left(0 - 2 + 24 - (0 + 3 + 8)\right)$

$= 3\left(-11 + 10\right) + \left(22 - 11\right) = -3 + 11 = \underline{\underline{8}}$

② $\log(1 + 2x^2 + 3y^4) + (x - y^2)^3 + \sin(x+y) = 0 \qquad (0,0)$

$F(x,y) = \log(1 + 2x^2 + 3y^4) + (x-y^2)^3 + \sin(x+y)$

- $F \in C^1(\mathbb{R}^2)$
- $F(0,0) = \log(1) + 0 + \cos 0 = 0$
- $\frac{\partial F}{\partial y}(x,y) = \frac{1}{1+2x^2+3y^4} \cdot 12y^3 + 3(x-y^2)^2 \cdot (-2y) + \cos(x+y)$

$\frac{\partial F}{\partial y}(0,0) = 0 + 0 + 1 \neq 0$

- $\frac{\partial F}{\partial x} = \frac{4x}{1+2x^2+3y^4} + 3(x-y^2)^2 + \cos(x+y)$

$\frac{\partial F}{\partial x}(0,0) = 0 + 0 + 1$

- $y'(0) = -\frac{1}{1} = \underline{\underline{-1}}$