## $05 \, \mathrm{Exam}$

Students have 120 minutes and can use any literature (notes, tables, textbooks...), but no technical devices (phone, calculator, watches...). Good luck!

## Example of the written part

1. (7 points) Compute the determinant of the following matrix

$$\begin{pmatrix} 1 & 0 & -1 & -4 \\ -2 & 3 & 0 & 1 \\ 2 & 0 & 3 & 4 \\ 1 & 1 & 2 & 3 \end{pmatrix}$$

2. (9 points) Show that

$$\log(1 + 2x^2 + 3y^4) + (x - y^2)^3 + \sin(x + y) = 0$$

determines at some neighborhood of the point [0,0] implicitly given function with variable x. Compute the first derivative of this function at the point [0,0].

3. (9 points)

Find global maximum and minimum of the function f on the set M.

$$f(x,y) = x^{2} + y^{2} + 16x - 12y, \qquad M = \{ [x,y] \in \mathbb{R}^{2}; x^{2} + y^{2} \le 25 \}.$$