① 7pt

$$\left(\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 2 & 0 & 1 & 0 \\ -1 & -1 & 2 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & -2 & 4 & 0 & 1 & 1 \\ 0 & -1 & 4 & 1 & 0 & 2 \end{array}\right) \sim \downarrow (-2)$$

$$\sim \left(\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & -1/2 & -1/2 \\ 0 & 0 & -4 & -2 & 1 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & -1/2 & -1/2 \\ 0 & 0 & 1 & 1/2 & -1/4 & 3/4 \end{array}\right) \sim \uparrow \cdot 2$$

$$\sim \left(\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & +1 \\ 0 & 0 & 1 & 1/2 & -1/4 & 3/4 \end{array}\right) \uparrow \cdot (-1) \sim \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 & -1 & +1 \\ 0 & 0 & 1 & 1/2 & -1/4 & 3/4 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1/2 & -1/2 \\ 0 & 1 & 0 & 1 & -1 & +1 \\ 0 & 0 & 1 & 1/2 & -1/4 & 3/4 \end{array}\right)$$

② 9pt (9 0)

$$(xy - 3x^2y^3)^2 = \sin y + 2\cos(x-y) - 2$$

$$F(x,y) = (xy - 3x^2y^3)^2 - \sin y - 2\cos(x-y) + 2$$

- $F \in C^1(\mathbb{R}^2)$
- $F(0,0) = (0-0)^2 - \sin 0 - 2\cos 0 + 2 = 0$
- $\frac{\partial F}{\partial y} = 2(xy - 3x^2y^3)(x - 3x^2 \cdot 3y^2) - \cos y + 2\sin(x-y)(-1)$

  $\frac{\partial F}{\partial y}(0,0) = 0 - \cos 0 + 2\sin 0 = -1 \neq 0$
- $\frac{\partial F}{\partial x} = 2(xy - 3x^2y^3)(y - 6xy^3) + 2\sin(x-y)$

  $\frac{\partial F}{\partial x}(0,0) = 0 + 0 = 0$
- $y'(0) = -\frac{0}{-1} = 0$

③

opt $\quad f = x^2 + 4y^2 \qquad M: \; x^2 + y^2 \leq 9$

- M is bounded and closed → compact
  f is continuous (polynomial) } f attains extrema

- $\text{int}M$

$\frac{\partial f}{\partial x} = 2x \qquad x = 0$

$\frac{\partial f}{\partial y} = 8y \qquad y = 0$

$[0,0] \in \text{int}M$

- ∂M
  - $g = x^2 + y^2 - 9$

    $\nabla g = (2x, 2y) \rightarrow (0,0) \notin \partial M$

  - Lagr. mult.

$2x + \lambda 2x = 0 \rightarrow 2x(1+\lambda) = 0$

$8y + \lambda 2y = 0$

$x^2 + y^2 = 9$

$x = 0 \qquad \lambda = -1$

$y = \pm 3 \qquad 8y - 2y = 0$

$[0, \pm 3] \qquad 6y = 0$

$y = 0 \rightarrow x = \pm 3$

$[\pm 3, 0]$

- Suspect points

$f(0,0) = 0 \rightarrow$ glob. min

$f(0,3) = 36$
$f(0,-3) = 36$ } → glob. max

$f(3,0) = 9$

$f(-3,0) = 9$