

①
7pt

$$\begin{vmatrix} 3 & -1 & -2 & 4 \\ 2 & 1 & 0 & 3 \\ -2 & -1 & 4 & 1 \\ 0 & 2 & 0 & -1 \end{vmatrix} = (-1)^{4+2} \cdot 2 \begin{vmatrix} 3 & -2 & 4 \\ 2 & 0 & 3 \\ -2 & 4 & 1 \end{vmatrix} + (-1)^{4+4} \cdot (-1) \begin{vmatrix} 3 & -1 & -2 \\ 2 & 1 & 0 \\ -2 & -1 & 4 \end{vmatrix}$$

$$= 2 \left(12 + 32 - (36 - 4) \right) - 1 \left(12 + 4 - (4 - 8) \right)$$

$$= 24 - 20 = 4$$

②
9pt

$$\arctan(x^2y+x) + \cos(x+y) = xy^2 \quad [1, -1]$$

$$F(x,y) = \arctan(x^2y+x) + \cos(x+y) - xy^2$$

$$F \in C^1(\mathbb{R}^2)$$

$$F(1, -1) = \arctan(-1+1) + \cos(1-1) - 1 = 0 + 1 - 1 = 0$$

$$\frac{\partial F}{\partial y} = \frac{1}{1+(x^2y+x)^2} \cdot x^2 + (-\sin(x+y)) - 2xy$$

$$\frac{\partial F}{\partial y}(1, -1) = 1 + (-\sin(0)) + 2 = 3 \neq 0$$

$$\frac{\partial F}{\partial x} = \frac{1}{1+(x^2y+x)^2} \cdot (2xy+1) + (-\sin(x+y)) - y^2$$

$$\frac{\partial F}{\partial x}(1, -1) = 1 \cdot (-2+1) + \sin(0) - 1 = -2$$

$$y'(1) = -\frac{-2}{3} = \frac{2}{3}$$

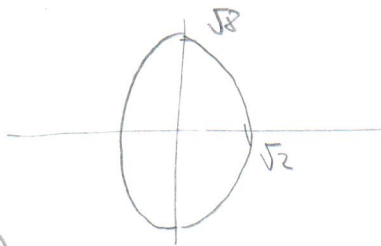
(3) $f(x,y) = xy$ $M: 4x^2 + y^2 \leq 8$

(a) M is ellipse

M is bounded and closed
 \rightarrow compact

f is continuous (polynomial)

$\therefore f$ attains extrema



(b) inter:

$$\frac{\partial f}{\partial x} = y$$

$$y = 0$$

susp. point: $(0,0)$

$$\frac{\partial f}{\partial y} = x$$

$$x = 0$$

(c) $\partial M:$

$$g = 4x^2 + y^2 - 8$$

$$\nabla g = (8x, 2y) = (0,0) \quad \text{for } (x,y) = (0,0) \notin \partial M$$

Lagrange multipliers

$$y + 2\lambda x = 0$$

$$\rightarrow y = -2\lambda x$$

$$x + 2\lambda y = 0$$

$$x + 2\lambda(-2\lambda x) = 0$$

$$4x^2 + y^2 = 8$$

$$x - 4\lambda^2 x = 0$$

$$x(1 - 4\lambda^2) = 0$$

$$x = 0$$

$$\downarrow$$

$$y = \pm\sqrt{8}$$

$$\lambda = \pm \frac{1}{4}$$

$$y + 2x = 0$$

$$y = -2x$$

$$4x^2 + 4x^2 = 8$$

$$x = \pm 1$$

$$y = \mp 2$$

$$[-1, -2] \quad [1, 2]$$

$$y - 2x = 0$$

$$y = 2x$$

$$4x^2 + 4x^2 = 8$$

$$x = \pm 1$$

$$y = \pm 2$$

$$[1, 2], [-1, -2]$$

(d)

$$f(0,0) = 0$$

$$f(1,2) = 2$$

$$f(-1,-2) = -2$$

} glob. max

$$f(1,-2) = -2$$

$$f(-1,2) = 2$$

} glob. min