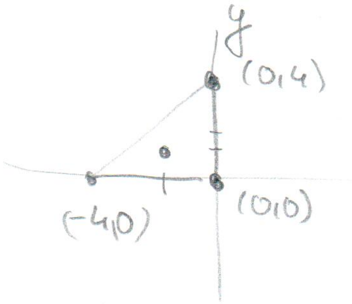
(3)

9pt

$f = x^2y - x^2 - 4y + 4x$ M: triangle: $[0,0]$, $[0,4]$, $[-4,0]$



- M is bounded and closed → compact
  f is cont. (polynomial)

1 } f attains extrema on M

- int M: $\frac{\partial f}{\partial x} = 2xy - 2x + 4$ $\quad 2xy - 2x + 4 = 0 \to [2,0] \notin M$

1+1,5 $\frac{\partial f}{\partial y} = x^2 - 4$ $\quad x^2 = 4 \quad x = \pm 2$ $\quad [-2,2] \in M$

- ∂M:

1 $y = 0, \quad x \in (-4,0)$:

$g_1(x) = f(x,0) = -x^2 + 4x \quad g_1' = -2x + 4 \to x = 2 \notin M$

1 $x = 0 \quad y \in (0,4)$

$f(0,y) = -4y \to$ extrema at vertices $[0,0]$, $[0,4]$

$y = x + 4 \quad x \in (-4,0)$

$g_2(x) = f(x, x+4) = x^2(x+4) - x^2 - 4(x+4) + 4x$

2 $= x^3 + 3x^2 - 16$

$g_2'(x) = 3x^2 + 6x = 3x(x+2)$ $\quad x_1 = 0 \quad y_1 = 4$
$x_2 = -2 \quad y_2 = 2$

- Suspect points:

1,5 $f(-2,2) = -12$

$f(0,0) = 0 \leftarrow$ glob. max

$f(0,4) = -16$

$f(-4,0) = -32 \leftarrow$ glob. min

(1)

7pt

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 2 & -3 & 1 \\ -4 & 1 & 1 \end{pmatrix}$$

1 5

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 2 & -3 & 1 & 0 & 1 & 0 \\ -4 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & -3 & 3 & -2 & 1 & 0 \\ 0 & 1 & -3 & 4 & 0 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & -3 & 3 & -2 & 1 & 0 \\ 0 & 0 & -6 & 10 & 1 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & -6 & 0 & +6 & 3 & 3 \\ 0 & 0 & -6 & 10 & 1 & 3 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1/2 & -1/2 \\ 0 & 0 & 1 & -5/3 & -1/6 & -1/2 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2/3 & -1/6 & -1/2 \\ 0 & 1 & 0 & -1 & -1/2 & -1/2 \\ 0 & 0 & 1 & -5/3 & -1/6 & -1/2 \end{array}\right)$$

1 A^{-1}

(2) $\ln(x^2+3y^2) + \sqrt{x^2+5y^2} = 3x^2+3y-2$ $[1,0]$

9pt

1 $F(x,y) = \ln(x^2+3y^2) + \sqrt{x^2+5y^2} - 3x^2 - 3y + 2$

1 • $F \in C^1(\mathbb{R}^2 \setminus \{0,0\})$

1 • $F(1,0) = \ln(1) + \sqrt{1} - 3 - 0 + 2 = 0$

1 • $\dfrac{\partial F}{\partial y} = \dfrac{1}{x^2+3y^2} \cdot 6y + \dfrac{1}{2}\dfrac{1}{\sqrt{x^2+5y^2}} \cdot 10y - 3$

1 + 1 at $(1,0)$: $\dfrac{0}{1} + \dfrac{1}{2} \cdot \dfrac{1}{1} \cdot 0 - 3 = -3 \neq 0$

1 + 1 • $\dfrac{\partial F}{\partial x} = \dfrac{2x}{x^2+3y^2} + \dfrac{1}{2}\dfrac{2x}{\sqrt{x^2+5y^2}} - 6x$ at $(1,0)$: $\dfrac{2}{1+0} + \dfrac{1}{1} - 6 = -3$

1 • Hence $y'(1,0) = -\dfrac{-3}{-3} = \underline{\underline{-1}}$