

## 02 Exam

Students have 120 minutes and can use any literature (notes, tables, textbooks...), but no technical devices (phone, calculator, watches...).

Good luck!

1. (7 points) Find the inverse of the following matrix

$$\begin{pmatrix} 1 & 0 & -1 \\ 2 & -3 & 1 \\ -4 & 1 & 1 \end{pmatrix}$$

2. (9 points) Show that

$$\ln(x^2 + 3y^2) + \sqrt{x^2 + 5y^2} = 3x^2 + 3y - 2$$

determines at some neighborhood of the point  $[1, 0]$  implicitly given function with variable  $x$ . Compute the first derivative of this function at the point  $[1, 0]$ .

*By mistake, the exam contained the following function (which does not satisfy conditions of the Implicit function theorem)*

$$\ln(x^2 + 3y^2) + \sqrt{x^2 + 5y^2} = 3xy^2 + 3y - 2$$

3. (9 points)

Find global maximum and minimum of the function  $f$  on the set  $M$ :

$$f(x, y) = x^2y - x^2 - 4y + 4x$$

and  $M$  is a triangle with vertices  $(0, 0)$ ,  $(0, 4)$  and  $(-4, 0)$ .

Hint: the triangle edges can be described as these segments:

- $y = x + 4, x \in [-4, 0]$ ,
- $y = 0, x \in [-4, 0]$ ,
- $x = 0, y \in [0, 4]$ .