① 7pt

$$\begin{vmatrix} -1 & 2 & 0 & -3 \\ 4 & 0 & -1 & -1 \\ 2 & 0 & 3 & 1 \\ -2 & -1 & 1 & 0 \end{vmatrix} = (-1)^{1+2} \cdot 2 \cdot \begin{vmatrix} 4 & -1 & -1 \\ 2 & 3 & 1 \\ -2 & 1 & 0 \end{vmatrix} + (-1)^{4+2} (-1) \begin{vmatrix} -1 & 0 & -3 \\ 4 & -1 & -1 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= -1 \cdot 2 \left(2 - 2 - (6+4) \right) - 1 \cdot \left(1 - 36 - (6+3) \right)$$

$$= 20 + 44 = 64$$

② 4pt

$$\underbrace{e^{xy} - \cos(x+y) + \sin(3x-y)}_{F(x,y)} = 0 \qquad a = [0,0]$$

(1) $F \in C^1(\mathbb{R}^2)$

(2) $F(0,0) = 1 - 1 + 0 = 0$

(3) $\frac{\partial F}{\partial y} = e^{xy} \cdot x + \sin(x+y) + \cos(3x-y) \cdot (-1)$

$\frac{\partial F}{\partial y}(0,0) = 0 + 0 - 1 \neq 0$

$\frac{\partial F}{\partial x} = e^{xy} \cdot y - \sin(x+y) + \cos(3x-y) \cdot 3$

$\frac{\partial F}{\partial x}(0,0) = 0 - 0 + 1 \cdot 3 = 3$

$y'(0) = -\frac{3}{-1} = \underline{\underline{3}}$

(3) $f(x,y) = x^2y + 8y$ $\quad$ $M: x^2 + 9y^2 \leq 1$

9pt

- M is ellipse 

bounded + closed = compact
f is continuous (polyn.) at $\mathbb{R}^2$ } f attains extrema

- int M: $\nabla f = (2xy, x^2+8) \neq (0,0)$

no critical points here on int M

∂M

- $g(x,y) = x^2 + 9y^2 - 1$

$\nabla g = (2x, 18y) = 0 \iff (x,y) = (0,0) \notin \partial M$

- Lagr. mult.

$$2xy + 2x\lambda = 0$$
$$x^2 + 8 + 18\lambda y = 0$$
$$x^2 + 9y^2 = 1$$

$\rightarrow$ $2x(y+\lambda) = 0$

$x = 0$ $\quad$ $y = -\lambda$

$x = 0$: $9y^2 = 1$, $y = \pm 1/3$, $[0, \pm 1/3]$

$y = -\lambda$:

$$x^2 + 8 - 18y^2 = 0$$
$$x^2 + 9y^2 = 1$$

$$8 - 27y^2 = -1$$
$$27y^2 = 9$$
$$y^2 = \frac{1}{3}$$
$$y = \pm\frac{1}{\sqrt{3}}$$

$$x^2 + 9 \cdot \tfrac{1}{3} = 1$$
$$x^2 + 3 = 1$$
$$x^2 = -2$$

impossible

- Conclusion $\rightarrow$ 2 suspect points $[0, \pm\frac{1}{3}]$

$f(0, 1/3) = \frac{8}{3}$ $\quad$ glob. max

$f(0 - 1/3) = -8/3$ $\quad$ glob. min