(1)

$$x' = y + t \qquad x(0) = \frac{3}{2}$$
$$y' = y + e^{2t} \qquad y(0) = 8$$

$$x' = 0x + 1y + t$$
$$y' = 0x + y + e^{2t}$$

$$\left(\begin{array}{cc|c} \lambda & -1 & t \\ 0 & \lambda-1 & e^{2t} \end{array}\right)$$

• $y' - y = e^{2t}$ $\qquad y_h = c_1 e^t$

$\lambda - 1 = 0$

$\lambda = 1$ $\qquad e^{2t}(1 \cdot \cos 0t + 0 \sin 0t)$

$y_p = Ae^{2t}$ $\qquad 2+0$ není kořen

$y_p' = 2Ae^{2t}$

$2Ae^{2t} - Ae^{2t} = e^{2t}$ $\qquad A = 1$

$y = c_1 e^t + e^{2t}$ $\qquad t, c_{1,2} \in \mathbb{R}$

• $x' = y + t = c_1 e^t + e^{2t} + t$ // přímo zintegrujeme

$x = c_1 e^t + \frac{1}{2} e^{2t} + \frac{1}{2} t^2 + c_2$

• podm. $y(0) = c_1 + 1 = 8$ $\qquad c_1 = 7$

$x(0) = c_1 + \frac{1}{2} + c_2 = \frac{3}{2}$ $\qquad 7 + \frac{1}{2} + c_2 = \frac{3}{2}$ $\qquad c_2 = -6$

• závěr: $\underline{y = 7e^t + e^{2t}, \quad x = 7e^t + \frac{1}{2} e^{2t} + \frac{1}{2} t^2 - 6}, \quad t \in \mathbb{R}$

(2) $$\lim_{(x,y) \to (0,0)} \frac{xy^2}{x^3 + y^3}$$

po přímkách: $x = y$,

$$\lim_{x \to 0} \frac{x^3}{x^3 + y^3} = \lim_{x \to 0} \frac{1}{2} = \frac{1}{2}$$

$y = 0$ $\qquad \neq$ $\qquad$ závěr $\lim_{(x,y) \to (0,0)} \frac{xy^2}{x^3+y^3}$ $\nexists$

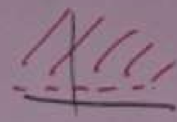
$$\lim_{x \to 0} \frac{0}{x^3 + 0} = 0$$

③ $f(x,y) = |x| \log y$

• D_f: $y>0$ 

$[x,y]\in\mathbb{R}^2$

• $\frac{\partial f}{\partial x} = \operatorname{sgn}(x)\cdot\log y$ $\quad x\neq 0,\ y>0$ 

$\frac{\partial f}{\partial y} = |x|\cdot\frac{1}{y}$ $\quad x\in\mathbb{R},\ y>0$ 

• pro body $[0,y]$, $y>0$

z definice

$$\frac{\partial f}{\partial x}(0,y) = \lim_{t\to 0}\frac{f(0+t,y)-f(0,y)}{t} = \lim_{t\to 0}\frac{|t|\log y - 0}{t}$$

$$= \lim_{t\to 0}\underbrace{\frac{|t|}{t}}\log y = \begin{cases}\nexists & \log y\neq 0\\ 0 & \log y = 0\end{cases}$$

$\rightarrow y=1$

$\lim_{t\to 0+} 1 = 1$

$\lim_{t\to 0-} -1 = -1$

• Závěr: $\frac{\partial f}{\partial y} = |x|\cdot\frac{1}{y}$ na D_f

$$\frac{\partial f}{\partial x} = \begin{cases}\operatorname{sgn}(x)\cdot\log y & x\neq 0 \quad y>0\\ 0 & (0,1)\\ \nexists & \text{jinak na } D_f\end{cases}$$