(1) $f_n(x) = \dfrac{2nx}{n^2 + x^2}$ na $\mathbb{R}$

• bod. lim: fix $x \in \mathbb{R}$

(1) $\lim\limits_{n\to\infty} \dfrac{n}{n^2} \cdot \dfrac{2x}{1 + \frac{x^2}{n^2}} = 0 \cdot \dfrac{2x}{1+0} = 0$ tedy $f \equiv 0$

• fix $n \in \mathbb{N}$:

(0,5) $\sigma_n = \sup \left\{ \left| \dfrac{2nx}{n^2+x^2} \right|, \ x \in \mathbb{R} \right\}$

(1) $\lim\limits_{x\to\pm\infty} \dfrac{x}{x^2} \dfrac{2n}{\frac{n^2}{x^2} + 1} = 0 \dfrac{2n}{0+1} = 0$

(2) $f_n' = \dfrac{2n(n^2+x^2) - 2nx \cdot 2x}{(n^2+x^2)^2} = \dfrac{2n^3 + 2nx^2 - 4nx^2}{(n^2+x^2)^2}$

$2n^3 - 2nx^2 = 0$

$n^3 = nx^2$

$n^2 = x^2$

$\pm n = x$

(1,5) • pak $\sigma_n = \dfrac{2n \cdot n}{n^2 + n^2} = 1 \not\to 0$

tedy $f_n \not\rightrightarrows 0$ na $\mathbb{R}$

(2) $\sum \dfrac{\sqrt[4]{n}\, x}{1 + n^3x^2}$ $x \in [0,\infty)$

• bod.: LSK s b_n $\dfrac{n^{1/4}}{n^3} = n^{\frac{1}{4}-3} = n^{-\frac{11}{4}}$ $\sum b_n$ K

$x \neq 0$ $\lim\limits_{n\to\infty} \dfrac{\frac{\sqrt[4]{n}\, x}{1+n^3x^2}}{\frac{\sqrt[4]{n}}{n^3}} = \lim\limits_{n\to\infty} \dfrac{n^3}{n^3} \cdot \dfrac{x}{\frac{1}{n^3} + x^2} = \dfrac{x}{0+x^2} = \dfrac{1}{x} \in (0,\infty)$

(1)

Tedy $\sum f_n$ K

Pro $x=0$ je $\sum 0$ K

$\Big\}$ $\sum f_n \to$ na $[0,\infty)$

(0,5) • fix $n \in \mathbb{N}$ $\sigma_n = \sup \left\{ \left| \dfrac{\sqrt[4]{n}\, x}{1+n^3x^2} \right|, \ x \in [0,\infty) \right\}$

(2) $f_n' = \sqrt[4]{n} \, \dfrac{1+n^3x^2 - x \cdot 2xn^3}{(1+n^3x^2)^2}$

$1 - n^3x^2 = 0$

$\dfrac{1}{n^3} = x^2$

$\pm \dfrac{1}{\sqrt{n^3}} = x$

$$f_n\left(\frac{1}{\sqrt{n^3}}\right) = \frac{\sqrt[4]{n}\cdot\frac{1}{n^{3/2}}}{1+n^3\cdot\frac{1}{n^3}} = \frac{1}{2}\, n^{\frac{1}{4}-\frac{3}{2}} = \frac{1}{2} n^{-5/4}$$

(1) $\lim_{x\to\infty} f_n = 0 \qquad \lim_{x\to 0+} f_n = 0$

Weierstrass

(1,5) $r_n = \frac{1}{2n^{5/4}} \qquad \sum \frac{1}{2n^{5/4}}$ k $\Rightarrow \sum f_n \rightrightarrows$ na $\mathbb{R}$

(3) $\sum_{n=1}^{\infty} (-1)^n \frac{\operatorname{arccot}(xn^2)}{n+x^2}$

- Dirichlet $a_n = (-1)^n$ má om. č. součty

(1) $b_n = \frac{1}{n+x^2}$ fix $x\in\mathbb{R}$, pak $\frac{1}{n+x^2}$ je nerostoucí (v n)

(1) $\frac{1}{n+1+x^2} \leq \frac{1}{n+x^2}$

$n+x^2 \leq n+1+x^2$ ✓

(1) $b_n \rightrightarrows 0$?

$r_n = \sup_{x\in\mathbb{R}} \left\{\left|\frac{1}{n+x^2}\right|\right\}$

$\frac{1}{n+x^2} \leq \frac{1}{n}$ ✓ $\qquad \lim r_n \leq \lim \frac{1}{n} = 0 \Rightarrow b_n \rightrightarrows 0$

tedy $\sum \frac{(-1)^n}{n+x^2} \rightrightarrows$ na $\mathbb{R}$

- Abel $b_n = \operatorname{arccot}(xn^2)$

(1) fix x: $\operatorname{arccot} xn^2 \begin{cases} \text{klesající pro} & x>0 \\ =\frac{\pi}{2} & x=0 \\ \text{rostoucí} & x<0 \end{cases}$

Navíc $0 \leq \operatorname{arccot} xn^2 \leq \pi \quad \forall x\in\mathbb{R}, \forall n\in\mathbb{N}$

(1)

$a_n = \sum \frac{(-1)^n}{n+x^2} \rightrightarrows$ na $\mathbb{R}$

(1)

$\Rightarrow \sum \frac{(-1)^n}{n+x^2} \operatorname{arccot}(n^2x) \rightrightarrows$ na $\mathbb{R}$