$$\int_1^\infty \underbrace{x^\alpha \sin x}_{f}\, dx, \quad \alpha \in \mathbb{R}$$

(a) $\alpha < -1$ Př. $\int_1^\infty \frac{1}{x^2} \sin x \, dx$

$\int_1^\infty |f|$ K ze SK: $|x^\alpha \sin x| \le x^\alpha$ $\int_1^\infty x^\alpha$ K $\alpha < -1$

AK, tedy K

(b.1) $\alpha \in [-1, 0)$ Př. $\int_1^\infty \frac{\sin x}{\sqrt{x}} \, dx$

z Dirichleta K $g = x^\alpha$ $f = \sin x$ $[1,\infty) = [a,b)$

f, g spoj na $[1,\infty)$ $F = -\cos x$ om.

g je om. 

monotónní, $\lim_{x\to\infty} x^\alpha = 0$

(b.2) AK:

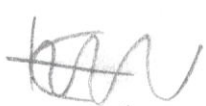
$|x^\alpha \sin x| \ge x^\alpha \sin^2 x = x^\alpha \cdot \frac{1}{2}(1 - \cos 2x)$

$\int_1^\infty \frac{1}{2} x^\alpha$ D $-\frac{1}{2}\int \frac{\cos 2x}{x^\alpha}$ K z Dirichleta $\to$ jako výše

K + D = D

Závěr: ze SK $\int_1^\infty |x^\alpha \sin x|$ D

Dohromady pro $\alpha \in [-1, 0)$ NAK

(c) $\alpha \ge 0$  BC-podm.

$$\left| \int_{k\pi}^{(k+1)\pi} \underbrace{x^\alpha \sin x}_{\ge 0} \, dx \right| \ge \int_{k\pi}^{(k+1)\pi} (k\pi)^\alpha \sin x \, dx = (k\pi)^\alpha \left[-\cos x\right]_{k\pi}^{(k+1)\pi} = (k\pi)^\alpha(-(-1) - (-1)$$

k sudé

$$= 2(k\pi)^\alpha \underset{k \ge 1}{\ge} 2\pi^\alpha$$

Zvolme $\varepsilon = 2\pi^\alpha$

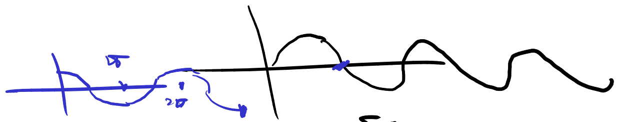
$\forall b' \in (1,\infty)$ zvolme $x_1 = k\pi$, $x_2 = (k+1)\pi$ k dost velké, sudé pak $\left|\int_{x_1}^{x_2} f\right| \ge 2\pi^\alpha = \varepsilon$ D

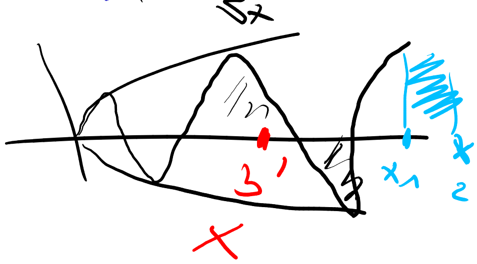
BC podm.

f spoj. na $[a,b)$.

$$\int_a^b f \quad D \iff \exists \varepsilon > 0 \quad \forall b' \in (a,b)$$
$$\exists x_1, x_2 \quad b' < x_1 < x_2 < b:$$
$$\left| \int_{x_1}^{x_2} f \right| \geq \varepsilon$$

Př. $\displaystyle\int_1^\infty x^\alpha \sin x \, dx \qquad \alpha \geq 0$





k sudé

$k\pi, (k+1)\pi$ $\qquad (\pi, 2\pi) \qquad 2\pi, 3\pi$

$$\left| \int_{k\pi}^{(k+1)\pi} \underbrace{x^\alpha \sin x}_{\geq 0} \, dx \right| \geq \int_{k\pi}^{(k+1)\pi} (k\pi)^\alpha \sin x \, dx =$$

$$= (k\pi)^\alpha \left[-\cos x \right]_{k\pi}^{(k+1)\pi} = (k\pi)^\alpha \left(-(-1) - (-1) \right) = 2(k\pi)^\alpha \underset{k \geq 1}{\geq} 2\pi^\alpha$$

Zvolme $\varepsilon = 2\pi^\alpha$ $\quad \forall b' \in (1, \infty)$

zvolme $x_1 = k\pi$ $\qquad$ k sudé, dost velké

$x_2 = (k+1)\pi$

pak $\left| \displaystyle\int_{x_1}^{x_2} f \right| \geq 2\pi^\alpha = \varepsilon$. $\qquad$ Z BC podm.

$\rightarrow$ divergence.