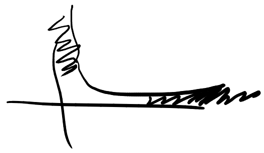
Neabsol. konv. $\int$

$$\int_0^\infty \underbrace{\frac{\arctan x^2}{x^a}\sin(2x)}_{f}\,dx \qquad a>0$$



• problémy: ∞, 0

$$\int_0^\infty = \int_0^{\pi/2} + \int_{\pi/2}^\infty$$

• $\int_0^{\pi/2}$: f nemění znaménko. AK splývá s K

LSK: $g(x) = \dfrac{x^2\cdot 2x}{x^a}$ $\qquad \int_0^{\pi/2} 2\,x^{3-a}\,dx$ K $\iff 3-a>-1$

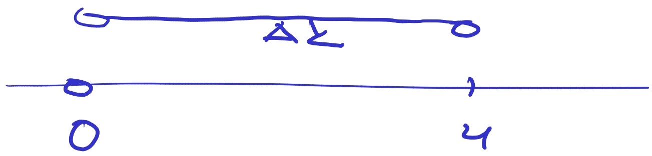
$\boxed{4>a}$

$g\geq 0$ ✓ $\qquad f, g$ spoj. $(0,\pi/2]$ ✓

$$\lim_{x\to 0+}\frac{f}{g} = \lim_{x\to 0+}\frac{\frac{\arctan x^2}{x^a}\sin(2x)}{\frac{x^2\cdot 2x}{x^a}} = \lim_{x\to 0+}\frac{\arctan x^2}{x^2}\cdot\frac{\sin 2x}{2x}$$

$$= 1 \in (0,\infty)$$

$\int_0^{\pi/2} f$ K $\iff \int_0^{\pi/2} g$ K $\iff \boxed{a<4}$

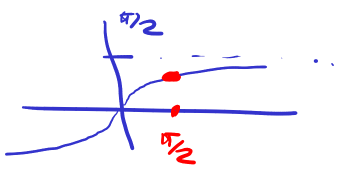


$$\int_{\pi/2}^\infty \frac{\arctan x^2}{x^a}\sin(2x)\,dx$$

• f mění znaménko

AK?: $\displaystyle\int_{\pi/2}^\infty \frac{\arctan x^2}{x^a}\,|\sin(2x)|\,dx$

$\odot \qquad \displaystyle\int_{\pi/2}^\infty \frac{|\sin(2x)|}{x^a}\,dx$



• SK

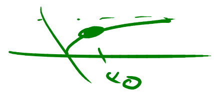
(podm.) $\quad \dfrac{\operatorname{arctan} x^2}{x^a} |\sin(2x)| \le \dfrac{\frac{\pi}{2} \cdot 1}{x^a}$ $\quad K \iff a > 1$

ze SS: $\displaystyle\int_{\pi/2}^{\infty}$ | | K $\boxed{a > 1}$

• $a \le 1$?

SK $\quad \dfrac{\operatorname{arctan} x^2}{x^a} |\sin 2x| \ge \dfrac{|\sin(2x)|}{x^a}$ D (tabulka)

$\operatorname{arctan} x^2 \ge 1$ pro $x \in (\frac{\pi}{2}, \infty)$ (od jistého x_0)



ze SS: $a \le 1$

$\displaystyle\int_{\pi/2}^{\infty}$ D

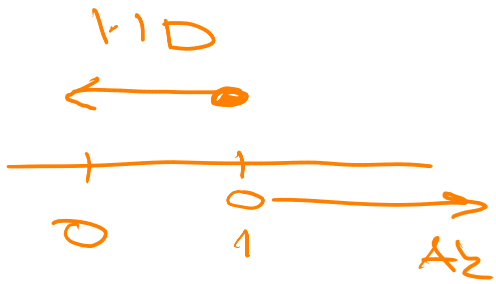
$$\int_{\pi/2}^{\infty} \frac{\sin(2x)}{x^a} = \int_{\pi}^{\infty} \frac{1}{2} \frac{\sin y}{\frac{1}{2^a} y^a} \, dy$$

$y = 2x$

$dy = 2\,dx$

$$= \frac{2^a}{2} \int_{\pi}^{\infty} \frac{\sin y}{y^a} \, dy$$

je v tabulce



• $a \in (0, 1]$ NAK ?

$$\int_{\pi/2}^{\infty} \frac{\operatorname{arctan} x^2}{x^a} \sin(2x) \, dx$$

$$\int_{\pi/2}^{\infty} \underbrace{\frac{1}{x^a}}_{g} \underbrace{\sin(2x)\,dx}_{f}$$

$\boxed{\text{Dir}}$

f, g spoj $[\pi/2, \infty)$ ✓

F je PF f, $F = -\frac{1}{2}\cos(2x)$ om.

$\lim_{x \to \infty} \frac{1}{x^a} = 0$ ✓ $\quad \frac{1}{x^a}$ monot. ✓

$$\int_{\pi/2}^{\infty} \frac{\sin(2x)}{x^a} \quad K$$

$\boxed{\text{Abel}}$ $$\int_{\pi/2}^{\infty} \underbrace{\operatorname{arctan} x^2}_{g} \underbrace{\frac{\sin 2x}{x^a}}_{f}$$

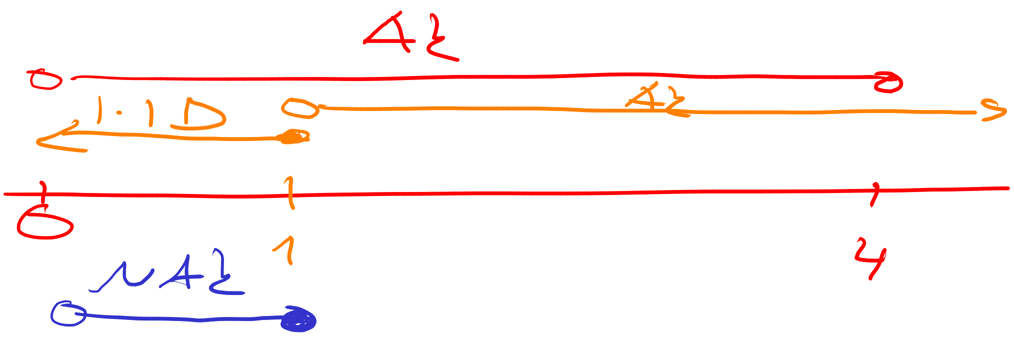
$|\operatorname{arctan} x^2| \leq \pi/2$ ✓

g omezená

$f \in N(\pi/2, \infty)$ ✓

$$\Rightarrow \int_{\pi/2}^{\infty} \operatorname{arctan} x^2 \frac{\sin(2x)}{x^a} \quad K$$

Závěr:



$a \in (1, 4)$ $\int$ AK

$a \in (0, 1]$ $\int$ NAK

$a \in [4, \infty)$ $\int$ D