Odmocniny

$t\neq -1$ $\quad t\neq 4$ $\quad \sqrt[3]{x+2}\neq -1$ $\quad \sqrt[3]{x+2}\neq 4$

$x+2\neq -1$ $\quad x+2\neq 64$

$x\neq -3$ $\quad x\neq 62$

$(-\infty,-3)$ $\quad (-3,62)$ $\quad (62,\infty)$

$$\int \frac{1}{\sqrt[3]{(x+2)^2}-3\sqrt[3]{x+2}-4}\,dx$$

2 vos.

$t=\sqrt[3]{x+2}$ $\qquad t^3-2=x \leftarrow \varphi(t)$

$3t^2\,dt = dx$ $\qquad \varphi' = 3t^2$ $\qquad t\neq 0$ ✓

$$\int \frac{1}{t^2-3t-4}\cdot 3t^2\,dt = \int 3 - \frac{3/5}{t+1} + \frac{48/5}{t-4}\,dt$$

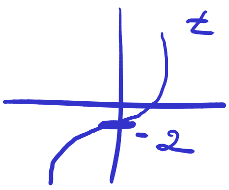
$t\neq -1$ $\quad t\neq 4$

$$\overset{c}{=} 3t - \frac{3}{5}\ln|t+1| + \frac{48}{5}\ln|t-4| =$$

$$= 3\sqrt[3]{x+2} - \frac{3}{5}\ln\left|\sqrt[3]{x+2}+1\right| + \frac{48}{5}\ln\left|\sqrt[3]{x+2}-4\right|$$

$x\in(a,b)$

$\varphi(t)$ $\quad t\in(\alpha,\beta)$ $\qquad (a,b)$



$(-\infty,-1) \xrightarrow[\text{na}]{\varphi} (-\infty,-3)$

$(-1,0) \xrightarrow{\varphi} (-3,-2)$

$(0,4) \longrightarrow (-2,62)$

$(4,\infty) \longrightarrow (62,\infty)$

v $x=-2$ slepování

1. vos

$$dt = \frac{1}{3}\,\frac{1}{\sqrt[3]{(x+2)^2}}\,dx$$

$$\int \frac{1}{\sqrt[3]{(x+2)^2}-3\sqrt[3]{x+2}-4}\cdot\frac{3\sqrt[3]{(x+2)^2}}{3\sqrt[3]{(x+2)^2}}\,dx$$

$$\int \frac{3t^2}{3t^2-3t-4}\,dt$$

$$\int \frac{1}{x^2}\sqrt{\frac{1+x}{x}}\,dx$$

$a_1 \quad b_1$

$x \in (-\infty, -1)$

$x \in (0, \infty)$

$a_2 \quad b_2$

$$t = \sqrt{\frac{1+x}{x}} \qquad t^2 = \frac{1+x}{x} \qquad xt^2 = 1 + x \qquad x(t^2-1) = 1$$

$$x = \frac{1}{t^2-1}$$

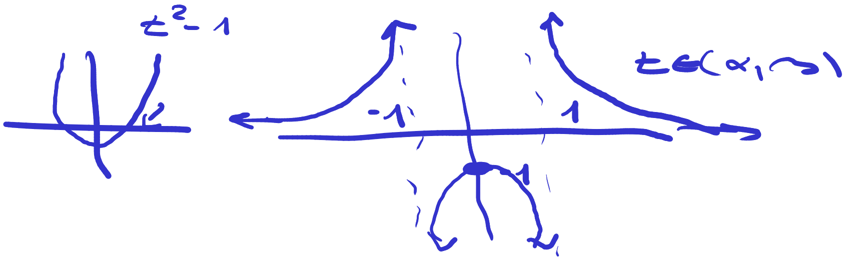
$$dx = -1 \cdot \frac{2t}{(t^2-1)^2}\,dt$$

$$\int \frac{1}{\left(\frac{1}{\cancel{t^2-1}}\right)^{\cancel{2}}} \cdot t\, \frac{-2t}{(\cancel{t^2-1})^2}\,dt = \int -2t^2\,dt \overset{c}{=} -\frac{2}{3}t^3$$

$$= -\frac{2}{3}\sqrt{\left(\frac{1+x}{x}\right)^3} \longrightarrow \begin{matrix} x \in (0, \infty) \\ x \in (-\infty, -1) \end{matrix}$$

$t \neq \pm 1$

$$\varphi(t) = \frac{1}{t^2-1} \qquad \varphi' = \frac{-2t}{(t^2-1)^2} \qquad t \neq 0$$



$\alpha_1 \quad \beta_1$ $\qquad a_1 \quad b_1$

$(0, 1) \xrightarrow{\varphi \text{ na}} (-\infty, -1)$

$(1, \infty) \xrightarrow{\varphi \text{ na}} (0, \infty)$

$a_2 \quad b_2$

$$\sqrt{(x+2)(x-3)} = \sqrt{\frac{x+2}{x-3}(x-3)^2}$$

$$= \sqrt{\frac{x+2}{x-3}} \cdot |x-3| \,.$$

$$\int \frac{1}{\sqrt{x^2+x+1}}\,dx \qquad x \in \mathbb{R}$$

$$t = \sqrt{x^2+x+1} - x \ge 0 \qquad \ge \sqrt{x^2}$$

$$t + x = \sqrt{x^2+x+1}$$

$$t^2 + 2tx + \cancel{x^2} = \cancel{x^2} + x + 1$$

$$x(2t-1) = 1 - t^2$$

$$x = \frac{1-t^2}{2t-1}$$

$$dx = -2\frac{t^2-t+1}{(1-2t)^2}\,dt$$

$$\int \frac{1}{t + \frac{1-t^2}{2t-1}} \cdot (-2)\frac{t^2-t+1}{(1-2t)^2}\,dt = \int \frac{-2(\cancel{t^2-t+1})}{\frac{\cancel{2t^2-t+1-t^2}}{2t-1}} \cdot \frac{1}{(1-2t)^2}\,dt$$

t^2-t+1

$$= \int -2\,\frac{-1}{1-2t}\,dt \overset{c}{=} \frac{2}{-2}\ln|1-2t| =$$

$$= -\ln\left|1 - 2\left(\sqrt{x^2+x+1} - x\right)\right| \qquad x \in (-\infty, \infty)$$

$$\varphi(t) = \frac{1-t^2}{2t-1} \qquad \varphi'(t) = -2\,\frac{t^2-t+1}{(1-2t)^2} \neq 0 \text{ vždy :)}$$

$t \neq \frac{1}{2}$

(α, β) $\qquad \alpha, \beta$ $\qquad (a, b)$

$\cancel{(-\infty, \frac{1}{2})} \longrightarrow \cancel{(-\infty,\infty)}$

$(\frac{1}{2}, \infty) \longrightarrow \underline{(-\infty, \infty)}$

α, β