$$\int \frac{x^4+2x^2+x+1}{(x-2)(x+1)^2}\,dx = \int x + \frac{5x^2+3x+1}{(x-2)(x+1)^2}\,dx$$

dělení $\qquad x \neq 2,\ x \neq -1$

$$(x^4+2x^2+x+1) : (x^3-3x-2) = x$$
$$-(x^4-3x^2-2x)$$
$$5x^2+3x+1$$

$$\int \frac{5x^2+3x+1}{(x-2)(x+1)^2}\,dx = \int \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{(x+1)^2}\,dx$$

$$5x^2+3x+1 = A(x+1)^2 + B(x-2)(x+1) + C(x-2)$$

$x = 2$ $\qquad 20+6+1 = A \cdot 9 \qquad A = 3$

$x = -1$ $\qquad 5-3+1 = -3C \qquad C = -1$

$x = 0$ $\qquad 1 = 3 - 2B + 2 \qquad B = 2$

Závěr

$$\int = \frac{x^2}{2} + 3\log|x-2| + 2\log|x+1| + \frac{1}{x+1}$$

$$x \in (-\infty,-1),\ (-1,2),\ (2,\infty)$$

$$\int \frac{x^3 - 2x^2 + 7x + 6}{(x^2+x+4)(x^2-3x+2)}\,dx = \int \underbrace{\frac{2}{x-2}}_{A} + \underbrace{\frac{-2}{x-1}}_{B} + \underbrace{\frac{1x-1}{x^2+x+4}}_{Cx+D}\,dx$$

$(x-2)(x-1)$ $\qquad x \neq 1, x \neq 2$

$$x^3 - 2x^2 + 7x + 6 = A(x-1)(x^2+x+4) + B(x-2)(x^2+x+4) + (Cx+D)(x-2)(x-1)$$

$$= \underline{Ax^3} + \underline{3Ax} - 4A + \underline{Bx^3} - \underline{3Bx^2} + \underline{2Bx} - 8B$$

$$+ \underline{Cx^3} - \underline{3Cx^2} + \underline{2Cx} + \underline{Dx^2} - \underline{3Dx} + 2D$$

x^3: $A + B + C = 1$

x^2: $-2 = -B - 3C + D$

x: $7 = 3A + 2B + 2C - 3D$

1: $6 = -4A - 8B + 2D$

$$\left(\begin{array}{cccc|c} A & B & C & D & \\ 1 & 1 & 1 & 0 & 1 \\ 0 & -1 & -3 & 1 & -2 \\ 3 & 2 & 2 & -3 & 7 \\ -2 & -4 & 0 & 1 & 3 \end{array}\right) \sim$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 1 \\ 0 & -1 & -3 & 1 & -2 \\ 0 & -1 & -1 & -3 & 4 \\ 0 & -2 & 2 & 1 & 5 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 3 & -1 & 2 \\ 0 & 0 & 2 & -4 & 6 \\ 0 & 0 & 4 & 7 & -3 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 3 & -1 & 2 \\ 0 & 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 15 & -15 \end{array}\right)$$

$D = -1 \quad C = 1 \quad B = -2 \quad A = 2$

Integrály

$$\int \frac{2}{x-2}\,dx \overset{c}{=} 2\log|x-2|$$

$$\int \frac{-2}{x-1}\,dx = -2\log|x-1|$$

$$\int \frac{x-1}{x^2+x+4}\,dx = \frac{1}{2}\int \frac{2x-2}{x^2+x+4}\,dx = \underbrace{\frac{1}{2}\int \frac{2x+1}{x^2+x+4}\,dx}_{\frac{1}{2}\log(x^2+x+4)} + \frac{1}{2}\int \frac{-3}{x^2+x+4}\,dx$$

$$\downarrow$$

$$-\frac{3}{2}\int \frac{1}{(x+\frac{1}{2})^2 + \frac{15}{4}}\,dx$$

$$\rightarrow \quad -\frac{3}{2}\int \frac{4}{15}\cdot \frac{1}{\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{15}}{2}}\right)^2+1}\,dx \quad = \quad -\frac{2}{5}\int \frac{1}{\left(\frac{2x+1}{\sqrt{15}}\right)^2+1}\,dx$$

$$= -\frac{2}{5}\cdot\frac{\sqrt{15}}{2}\arctan\left(\frac{2x+1}{\sqrt{15}}\right)$$

Zurück

$$\int \overset{c}{=} 2\log|x-2| - 2\log|x-1| - \frac{\sqrt{15}}{5}\arctan\left(\frac{2x+1}{\sqrt{15}}\right)$$

$$x \in (-\infty,1),\ (1,2),\ (2,\infty)$$