(1) $\sum \underbrace{\dfrac{\left(1+\frac{2}{n}\right)^{n^2}}{1+3^n}}_{a_n \geq 0}$

Cauchy $\lim\limits_{n\to\infty} \sqrt[n]{\dfrac{\left(1+\frac{2}{n}\right)^{n^2}}{1+3^n}} = \lim \dfrac{\left(1+\frac{2}{n}\right)^n}{\sqrt[n]{1+3^n}} = \dfrac{e^2}{3} > 1$, tedy $\sum a_n$ D

- $3 = \sqrt[n]{3^n} \leq \sqrt[n]{1+3^n} \leq \sqrt[n]{2\cdot 3^n} = \sqrt[n]{2}\cdot 3 \to 1\cdot 3$ 2 policajti

- $x_n = n$, $n \to \infty$, $n \neq \infty$ $\forall n \in \mathbb{N}$

$\lim\limits_{x\to\infty} \left(1+\frac{2}{x}\right)^x = \lim e^{x\log\left(1+\frac{2}{x}\right)} = e^2$ VOLSF, (S) e^y spoj v 2

$\lim\limits_{x\to\infty} \dfrac{\log\left(1+\frac{2}{x}\right)}{\frac{1}{x}\cdot 2}\cdot 2 \overset{4L}{=} 1\cdot 2$ VOLSF (P) $\frac{2}{x} \neq 0$ na [illegible]

(2) $\sum \underbrace{\dfrac{n+1}{n^2+3}}_{b_n} \underbrace{\cos\left(n\sqrt{\pi}\right)}_{a_n}$

$\sum a_n$ má om. č. součty

$\lim \dfrac{n+1}{n^2+3} = \lim \dfrac{n}{n^2}\cdot\dfrac{1+1/n}{1+3/n^2} \overset{4L}{=} 0\cdot\dfrac{1+0}{1+0} = 0$.

$f(x) = \dfrac{x+1}{x^2+3}$ $x \in \mathbb{R}$ $f' = \dfrac{x^2+3-(x+1)2x}{(x^2+3)^2} = \dfrac{-x^2-2x+3}{(x^2+3)^2} = \dfrac{-(x+3)(x-1)}{(x^2+3)^2}$

$f' < 0$ pro $x > 1$

tedy f je klesající na $(1,\infty)$

tedy b_n je klesající od $n = 2$

z Dirichleta $\sum a_n b_n$ konverguje

(3) $\int e^{2x} \sin(e^x)\,dx \overset{c}{=} -e^x\cos e^x + \sin e^x, \quad x \in (-\infty, \infty)$

$y = e^x$

$dy = e^x\,dx$

$\int y \sin y \, dy = -y\cos y + \int \cos y\, dy$

$u = y \qquad v' = \sin y \qquad \overset{c}{=} -y\cos y + \sin y$

$u' = 1 \qquad v = -\cos y$

$\varphi(x) = e^x$

$\varphi'(x) = e^x$

$f(y) = y \sin y \qquad x \in (\alpha, \beta) = (-\infty, \infty) \qquad y \in (a,b) = (-\infty, \infty)$

$\varphi(\alpha,\beta) = (0, \infty) \subseteq (a,b)$ ✓

(4) (a) $\sum a_{2n}$ k, $\sum a_{2n+1}$ k $\Rightarrow \sum a_n$ k

Platí. Uvažujme pol. část. součtů.

$t_{2N} = \sum_{n=1}^{N} a_{2n} \qquad u_{2N+1} = \sum_{n=0}^{N} a_{2n+1}$

Máme $\lim_{N\to\infty} t_{2N} = T \in \mathbb{R}$, $\lim_{N\to\infty} u_{2N+1} = U \in \mathbb{R}$

Pak pol. č. součtů $s_M = \sum_{n=1}^{M} a_n$

lze psát jako $s_M = u_{2N-1} + t_{2N} \to U + T \in \mathbb{R}$

pro $M = 2N$ sudé

pro $M = 2N+1$ liché $\quad s_M = u_{2N+1} + t_{2N} \to U + T \in \mathbb{R}$

$\Rightarrow \lim_{M\to\infty} s_M = U + T \in \mathbb{R}$ a $\sum a_n$ k

(b) $\sum a_n$ k $\Rightarrow \sum a_{2n}$ k a $\sum a_{2n+1}$ k

Neplatí. Protipříklad $\sum \frac{(-1)^n}{n}$ k, ale $\sum \frac{(-1)^{2n}}{2n} = \sum \frac{1}{2n}$ D

a $\sum \frac{(-1)^{2n+1}}{2n+1} = \sum \frac{-1}{2n+1}$ D.