(1) $\sum\limits_{n=1}^{\infty} \underbrace{\frac{n+\sqrt{n}}{\sqrt{1+2^n}}}_{a_n > 0}$

(1)

d'Al

(1) $\lim\limits_{n\to\infty} \dfrac{\dfrac{n+1+\sqrt{n+1}}{\sqrt{1+2^{n+1}}}}{\dfrac{n+\sqrt{n}}{\sqrt{1+2^n}}} = \lim \dfrac{n+1+\sqrt{n+1}}{n+\sqrt{n}} \cdot \sqrt{\dfrac{1+2^n}{1+2^{n+1}}}$

(1)

(2) $= \lim \dfrac{n}{n} \cdot \dfrac{1+\frac{1}{n}+\sqrt{1/n+1/n^2}}{1+\frac{1}{\sqrt{n}}} \sqrt{\dfrac{2^n}{2^n}} \sqrt{\dfrac{1/2^n+1}{1/2^n+2}} \overset{AL}{=} \dfrac{1+0+\sqrt{0}}{1+0} \cdot \sqrt{\dfrac{0+1}{0+2}} = \dfrac{1}{\sqrt{2}} < 1$

(1) Tedy z d'Al $\sum a_n$ k.

limity: Heine $x_n = \frac{1}{n} \to 0+$ $\quad \frac{1}{n} \neq 0$

$\lim\limits_{x\to 0+} \sqrt{x+x^2} = 0$ (spojitost) $\to \lim\limits_{n\to\infty} \sqrt{1/n+1/n^2} = 0$

Heine $x_n = 1/2^n \to 0+$ $\quad \frac{1}{2^n} \neq 0$

$\lim\limits_{x\to 0} \sqrt{\dfrac{x+1}{x+2}} = \sqrt{1/2}$ (spoj. tost) $\to \lim\limits_{n\to\infty} \sqrt{\dfrac{1/2^n+1}{1/2^n+2}} = \sqrt{1/2}$

(2) $\sum\limits_{n=1}^{\infty} (-1)^n \underbrace{\frac{\log n}{n}}_{b_n}$

škála (nebo Heine a LH)

(1) Leibniz $\lim\limits_{n\to\infty} \dfrac{\log n}{n} \overset{\swarrow}{=} 0$ (1)

monotonie: $f(x) = \dfrac{\log x}{x}$ $\quad x \in (0,\infty)$

(3) $f' = \dfrac{\frac{1}{x}x - \log x}{x^2} = \dfrac{1-\log x}{x^2}$ $\qquad 1-\log x < 0$ pro $x > e$

Tedy f je klesající na $[e,\infty)$

tedy b_n je klesající pro $n \geq 3$

(1) Tedy $\sum (-1)^n \frac{\log n}{n}$ k

(3) $\int (1+\sqrt{x}) \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx \overset{c}{=} 2\sqrt{x}e^{\sqrt{x}} \quad x \in (0,\infty)$
①

①

Substituce
$y = \sqrt{x}$

① $dy = \frac{1}{2\sqrt{x}} dx$

$\int 2(1+y)\, e^y = 2(1+y)e^y - \int 2e^y dy$

pp ① $u = 2(1+y) \quad v' = e^y \quad \overset{c}{=} 2\left((1+y)e^y - e^y\right)$

$u' = 2 \quad v = e^y \quad = 2ye^y$

Substituce ② $\varphi(x) = \sqrt{x} \quad x \in (\alpha, \beta) = (0,\infty) \quad \varphi(0,\infty) = (0,\infty)$

$\varphi' = \frac{1}{2\sqrt{x}} \quad \subseteq (-\infty,\infty)$ ✓

$f = 2(1+y)e^y \quad y \in (a,b) = (-\infty, \infty)$.

(4) (a) $\sum a_n$ AK, $\sum b_n$ AK $\Rightarrow \sum a_n + b_n$ AK

3×1

Platí $\sum |a_n + b_n| \le \sum |a_n| + \sum |b_n|$

konvergují z linearity $\sum$

tedy ze SK konverguje i $\sum |a_n + b_n|$. Tedy $\sum a_n + b_n$ je AK.

(b) $\sum a_n + b_n$ AK $\Rightarrow \sum a_n$ AK a $\sum b_n$ AK

Neplatí Uvažujme $a_n = n \quad b_n = -n$

Pak $\sum n - n = \sum 0$ ale $\sum n$ D a $\sum -n$ D.

(c) $\sum a_n$ AK, $\sum b_n$ AK $\Rightarrow \sum a_n b_n$ AK

Platí. $\sum |a_n|$ K, tedy $\lim_{n\to\infty} |a_n| = 0$ (NP).

Odtud $\exists M \ge 0: \quad |a_n| \le M$ (věta z 1. semestru)

Pak $\sum |a_n b_n| = \sum |a_n| \cdot |b_n| \le \sum M |b_n| = M \sum |b_n|$ K
(linearita $\sum$)

Tedy ze SK $\sum |a_n b_n|$ K tedy $\sum a_n b_n$ AK