AK $\sim \int$

$$\int_a^b |f(x)|\,dx \overset{?}{=} \infty \quad ?$$

$$\sum_{n=1}^{\infty} n^{\alpha} \qquad \alpha < -1 \text{ K} \qquad \alpha \geq \text{ D}$$

$$\int_1^{\infty} x^{\alpha} = \begin{cases} \alpha = -1 & [\ln|x|]_1^{\infty} = \infty \text{ D} \\ \alpha \neq -1 & \left[\dfrac{x^{\alpha+1}}{\alpha+1}\right]_1^{\infty} = \begin{cases} \alpha+1>0 & = \infty \text{ D} \\ \alpha+1<0 & = 0 - \dfrac{1}{\alpha+1} \text{ K} \end{cases} \end{cases}$$

$$\alpha < -1 \text{ K} \qquad \alpha \geq \text{ D}$$

$$\int_0^1 x^{\alpha} \qquad \alpha \leq -1 \text{ D} \qquad \alpha > -1 \text{ K}$$

$$\int_0^{\infty} \frac{1}{1+x^2}\,dx$$

$$\sum \frac{1}{1+n^2}$$

LSK $\frac{1}{n^2}$ & $\frac{1}{n^2}$

$$\int_0^{\infty} \underbrace{\frac{1}{1+x^2}}_{f}\,dx = [\arctan x]_0^{\infty} = \frac{\pi}{2}$$

- ∞ - v 0 f spoj.

- f spoj na om. uz?

  $[0,1]$ ✓ (na $[1,\infty)$ ne :))

- $\int_0^{\infty} = \int_{0}^{1}{}_{\text{K}} + \int_1^{\infty}{}_{\text{K}} \longrightarrow \int_0^{\infty} f$ K

  $\hookrightarrow$ spoj na om. uz $[0,1] \to$ K

• $\int_1^\infty \frac{1}{1+x^2}\,dx$

LSK $\quad g(x) = \frac{1}{x^2}$ $\qquad\qquad \int_1^\infty \frac{1}{x^2}\,dx$ k

$$\lim_{x\to\infty} \frac{f}{g} = \lim_{x\to\infty} \frac{\frac{1}{1+x^2}}{\frac{1}{x^2}} = 1 \in (0,\infty)$$

$\int_1^\infty f$ k $\iff \int_1^\infty g$ k

$\to \int_1^\infty f$ k :)

f, g spoj.: $[1,\infty)$ ✓

$\frac{1}{x^2} > 0$ $\qquad$ —"— ✓

• SK

$\int_1^\infty \frac{1}{1+x^2}$ k

$0 \le \frac{1}{1+x^2} \le \frac{1}{x^2}$ $\qquad \int_1^\infty \frac{1}{x^2}$ k

f spoj. na $[1,\infty)$ ✓

—

$\int_0^\infty \frac{1}{1+x^2}\,dx$ $\qquad$ f spoj. $[0,\infty)$

LSK $\int_0^\infty \underbrace{\frac{1}{x^2}}_{g(x)} = \left[-\frac{1}{x}\right]_0^\infty = 0 - (-\infty) = \infty$

$\lim_{x\to\infty} \frac{f}{g} = 1$ $\qquad \int f$ k $\iff \int g$ k

$\frac{1}{x^2}$ spoj $[0,\infty)$? $\times$

$$\int_0^{\pi/2} \frac{\cos x}{\underbrace{(\sin x)(\operatorname{arctan} x)^2}} \, dx$$

f spoj na $(0, \frac{\pi}{2}]$

g spoj na $(0, \pi/2]$ $\qquad$ $g \geq 0$ na $(0, \frac{\pi}{2}]$

$\cos x \to$

$\sin x \to \quad \cdot \quad \dfrac{1}{x \cdot x^2} = \dfrac{1}{x^3} = g(x)$

$(\operatorname{arctan} x)^2 \to$

$$\lim_{x \to 0+} \frac{\frac{\cos x}{\sin x (\operatorname{arctan} x)^2}}{\frac{1}{x \cdot x^2}} = \lim_{x \to 0+} \cos x \cdot \frac{x}{\sin x} \cdot \frac{x^2}{\operatorname{arctan}^2 x}$$

$= 1 \in (0, \infty)$

$\int f \ K \iff \int g \ K$ $\qquad$ $\int_0^{\pi/2} \frac{1}{x^3}$ D

závěr: $\int f$ D.