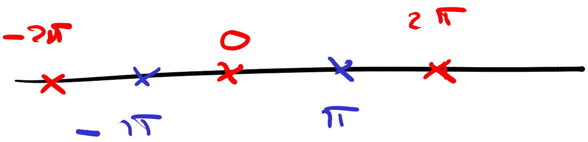
$$\int \frac{\sin x}{1-\cos^2 x}\,dx \qquad \text{cíl } (0,\pi)+k\pi$$

- $\cos x \neq 1$ $\qquad x \neq 0 + 2k\pi$
  $\neq -1$ $\qquad x \neq \pi + 2k\pi$



- (1) $\dfrac{\sin x}{1-(-\cos x)^2} = \dfrac{\sin x}{1-\cos^2 x} \overset{2}{\neq} -\dfrac{\sin x}{1-\cos^2 x}$

(2) $\dfrac{(-\sin x)}{1-\cos^2 x} \overset{2}{=} -\dfrac{\sin x}{1-\cos^2 x}$ ✓

$\longrightarrow$ $t = \cos x$.

(3) $\dfrac{-\sin x}{1-(-\cos x)^2} = \dfrac{-\sin x}{1-\cos^2 x} \overset{2}{\neq} \dfrac{\sin x}{1-\cos^2 x}$

$t = \cos x \qquad dt = -\sin x\, dx$

$$\int \frac{-1}{1-\cos^2 x}(-\sin x)\,dx = \int \overbrace{\frac{-1}{1-t^2}}^{f(t)}\,dt.$$

$$\overset{e}{=} -\frac{1}{2}\ln\left|\frac{1+t}{1-t}\right| = -\frac{1}{2}\ln\left|\frac{1+\cos x}{1-\cos x}\right|$$

$\varphi = \cos x \qquad (\alpha,\beta) = (0,\pi)+k\pi$

$f(t) = \dfrac{-1}{1-t^2}$

$\cancel{(-\infty,-1)}$
$(-1,1) = (a,b)$
$\cancel{(1,\infty)}$

$\varphi(\alpha,\beta) \subseteq (a,b) \qquad \cos(0,\pi) + k\pi = (-1,1)$

Výsledek je na $(\alpha,\beta). = (0,\pi)+k\pi$

$$\int \frac{1}{\sin^2 x + 2\cos^2 x}\,dx$$

- cil: $x \in \mathbb{R}$
- test: $\dfrac{1}{(-\sin x)^2 + 2(-\cos x)^2} = \dfrac{1}{\sin^2 x + 2\cos^2 x}$

$\rightarrow$ $\underline{t = \tan x}$ $\qquad dt = \dfrac{1}{\cos^2 x}\,dx$

1. VOS

- $$\int \frac{\cos^2 x}{\sin^2 x + 2\cos^2 x} \cdot \underbrace{\frac{1}{\cos^2 x}\,dx}$$

$$= \int \frac{1}{\frac{t^2}{1+t^2} + \frac{2\cdot 1}{1+t^2}} \cdot \underbrace{\frac{1}{1+t^2}\,dt}$$

$f(t)$ $\longrightarrow$

$$= \int \frac{1}{2+t^2}\,dt = \int \frac{1}{2} \cdot \frac{1}{1+\left(\frac{t}{\sqrt{2}}\right)^2}\,dt$$

$$\overset{c}{=} \frac{1}{2}\cdot\sqrt{2}\arctan\frac{t}{\sqrt{2}} = \frac{\sqrt{2}}{2}\arctan\left(\frac{\tan x}{\sqrt{2}}\right)$$

(α,β) $\longrightarrow$

- $\varphi = \tan x \qquad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) + k\pi$
- $f(t) \qquad t \in \mathbb{R} = (a,b)$
- $\varphi(\alpha,\beta) = \mathbb{R} \subseteq (a,b)$ ✓ PLUH

Výsledek: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) + k\pi$