$$\int \frac{1}{1+\sqrt{x}}\,dx$$

1. Vos

$$\int \frac{1}{1+\sqrt{x}} \cdot \frac{2\sqrt{x}}{2\sqrt{x}}\,dx = \int \frac{1}{1+y}\,2y\,dy = 2\int \frac{y+1-1}{y+1}\,dy$$

$y = \sqrt{x}$

$dy = \frac{1}{2\sqrt{x}}\,dx$

$$= 2\int 1 - \frac{1}{y+1}\,dy \overset{c}{=} 2y - 2\log|1+y|$$

$$= 2\sqrt{x} - 2\log(1+\sqrt{x})$$

$x \in (0,\infty)$
α, β

$\varphi(x) = \sqrt{x}$

$\varphi' = \frac{1}{2\sqrt{x}}$

$\varphi(0,\infty) = (0,\infty) \subseteq (-1,\infty)$

$y \in (-1,\infty)$
(a,b)

$f(y) = \frac{2y}{1+y}$ $\quad F(y) = 2y - 2\log|1+y|$

2. Vos

$$\int \frac{1}{1+\sqrt{x}}\,dx = \int \frac{1}{1+t}\,2t\,dt \overset{c}{=} 2t - 2\log|1+t|$$

$x = t^2 \quad (t = \sqrt{x})$

$$= 2\sqrt{x} - 2\log(1+\sqrt{x})$$

$dx = 2t\,dt$

$f(x) = \frac{1}{1+\sqrt{x}}$

$$\int f(\varphi(t))\cdot\varphi'(t)\,dt = \int \frac{1}{1+\sqrt{t^2}}\,2t\,dt \overset{c}{=} \underbrace{2t - 2\log|1+t|}_{G(t)}$$

$\varphi(t) = t^2$

$\varphi'(t) = 2t$

$\varphi^{-1} = \sqrt{x}$

$t \in (\alpha,\beta) = (0,\infty)$

$\varphi(0,\infty) = (0,\infty) = (a,b)$

$x \in (0,\infty)$

$\varphi' \neq 0$ na $(0,\infty)$

$G(\varphi^{-1}(x)) = 2\sqrt{x} - 2\log(1+\sqrt{x})$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-\sin^2 t}} \cos t\,dt = \int \frac{\cos t}{\sqrt{\cos^2 t}}\,dt$$

$\sin t = x$

$\cos t\,dt = dx$

$\arcsin x = t$

$$= \int \frac{\cos t}{|\cos t|} \overset{!}{=} \int \frac{\cos t}{\cos t}\,dt = \int 1\,dt$$

$$\overset{c}{=} t = \arcsin x$$

2VoS

$\varphi(t) = \sin t$ $\qquad f(x) = \dfrac{1}{\sqrt{1-x^2}}$

$\varphi'(t) = \cos t$

$\varphi^{-1}(x) = \arcsin x$ $\qquad f(\varphi(t))\,\varphi'(t) = \dfrac{1}{\sqrt{1-\sin^2 t}} \cos t$

(α, β)

$t \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ $\qquad G = t$

$x \in (-1, 1) = (a, b)$ $\qquad G(\varphi^{-1}(x)) = \arcsin x$

$\varphi\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) = (-1, 1)$

$(\alpha, \beta) \qquad (a, b)$

$\varphi' = \cos t \neq 0$ na (α, β)