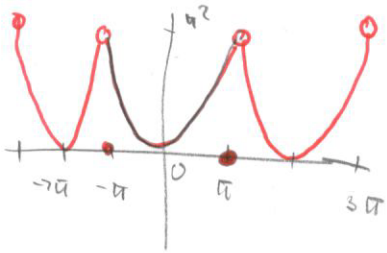
f 2π-periodic

$$f(x) = \begin{cases} x^2, & x \in (-\pi, \pi), \\ 0, & x = \pi. \end{cases}$$



(1) $BV(f, [-\pi, \pi]) = \pi^2 + \pi^2 + \pi^2 + \pi^2 = 4\pi^2$

(3) nebo: f je monot. na $(-\pi, 0)$ a $[0, \pi)$ → tedy je $BV(-\pi, \pi)$

(18) (2) ② f je sudá → $b_n = 0$

③ $$a_0 = 2 \cdot \frac{1}{\pi} \int_0^{\pi} x^2 dx = \frac{2}{\pi} \left[\frac{x^3}{3} \right]_0^{\pi} = \frac{2\pi^2}{3}$$

$n \in \mathbb{N}$: $$a_n = \frac{2}{\pi} \int_0^{\pi} x^2 \cos(nx)\, dx = \frac{2}{\pi} \left[x^2 \cdot \frac{1}{n} \sin(nx) \right]_0^{\pi} - \frac{2}{\pi} \int_0^{\pi} 2x \cdot \frac{1}{n} \sin(nx)\, dx$$

① u, v'; $u' = 2x$, $v = \frac{1}{n}\sin(nx)$; $u' = 2$, $v = -\frac{1}{n^2}\cos(n$

(5×2)

$$= -\frac{2}{\pi} \left[-\frac{2x}{n^2} \cos(nx) \right]_0^{\pi} + \frac{2}{\pi} \int_0^{\pi} \frac{-2}{n^2} \cos nx\, dx =$$

$$= +\frac{4}{\pi n^2} \pi (-1)^n - \frac{4}{\pi n^2} \left[\frac{\sin(nx)}{n} \right]_0^{\pi} = \frac{4(-1)^n}{n^2}$$

② $$S^f = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2} \cos(nx)$$

(7) (3) ① pro $x = 0$ ④ $f \in C^1(-\pi, \pi)$

② $f(0) = 0 = \frac{\pi^2}{3} + \sum \frac{4(-1)^n}{n^2} \cos(0)$ $\quad \Sigma = \frac{-\pi^2}{12}$

③ $$-\frac{\pi^2}{3} \frac{1}{4} = \sum \frac{(-1)^n}{n^2}$$